Possibilistic networks: A new setting for modeling preferences

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Abstract. Possibilistic networks are the counterpart of Bayesian networks in the possibilistic setting. There exist two types of Bayesian nets depending if a qualitative or a quantitative conditioning is used. Possibilistic nets have only been studied and developed from a reasoning-under-uncertainty point of view until now. In this short note, for the first time, one advocates their interest in preference modeling. Beyond their graphical appeal, they can be shown as providing a natural encoding of preferences agreeing with the inclusion-based partial order applied to the subsets of preferences violated in the different situations. Moreover they do not encounter the limitations of CP-nets in terms of representation capabilities. They also enjoy a logical counterpart that may be used for consistency checking. This short note provides a comparative discussion of the merits of possibilistic networks with respect to other existing preference modeling frameworks.

1 Introduction

Preferences are usually expressed by means of pieces of information in a local manner, rather than as a complete preorder between the different possible states of the world. This state of facts has led AI researchers to propose compact representation formats for preferences and procedures for computing a plausible ranking between completely described situations from such representations, in the last fifteen years. Conditional preference networks [5] (CP-nets for short) have emerged as a popular reference setting for representing preferences, leading to different refinements [4, 13], as well as some alternative approaches [3, 7, 11]. See [6] for a brief overview. Inspired from Bayesian nets, CP-nets inherit their graphical nature, and besides, rely on a simple, apparently natural principle, named *ceteris paribus*, which allows to extend any contextual preference "in context c, I prefer a to $\neg a$" (denoted for short $c : a \succ \neg a$), to any particular specification b of the other variables used for describing the considered situations, i.e., the preference is understood as $\forall b, cab$ is preferred to $c\neg ab$.

The CP-net approach perfectly exemplifies the ingredients needed for a satisfactory completion of preferences, stated in a possibly conditional manner, into a preorder useful for a user: i) a simple representation setting, preferably having a graphical counterpart for elicitation ease, ii) a natural principle for making explicit the preferences between completely described situations, and iii) an algorithm for determining how to compare two complete situations according to the existence of a path of worsening flips linking them. In spite of their appealing features, CP-nets have some limitations. First, there exist preorders that make sense and for which there does not exist any CP-net that can be associated to them. Moreover, they tend to force some debatable priorities between preferences associated to nodes in the CP-nets, beyond what is really expressed by the preferences one starts with [9, 10].

In this short paper, we advocate possibilistic networks as a valuable tool for representing preferences. First, possibilistic nets are the counterpart of Bayesian nets in possibility theory, based on a possibilistic Bayesian-like conditioning rule. Although they have been only used for uncertainty modeling until now, they can serve preference modeling purposes as well, as shown in the following, without having the CP-nets limitations mentioned above. The paper is organized as follows. Section 2 provides a brief background on possibilistic networks. Then Section 3 proposes and explains their use in preference modeling and establishes some properties. The paper ends with a short discussion comparing CP-nets and the preference possibilistic network.

2 Possibilistic networks

Let $V = \{V_1, \ldots, V_N\}$ be a set of N variables. Each variable V_i has a value domain $D(V_i)$. v_i denotes any value of V_i. $\Omega = \{\omega_1, \ldots, \omega_i\}$ denotes the universe of discourse, which is the Cartesian product of all variable domains in V. Each element $\omega_i \in \Omega$ is called an interpretation. We start by a brief recall of possibility theory [8, 14] which relies on the idea of a possibility distribution π, which is a mapping from a universe of discourse Ω to the unit interval $[0, 1]$, or to any bounded totally ordered scale. This possibilistic scale could be interpreted in twofold: a numerical interpretation when values have a real sense and an ordinal one when values only reflect a total preorder between the different interpretations. $\pi(\omega_i) = 0$ means that ω_i is fully impossible, while $\pi(\omega_i) = 1$ means that ω_i is fully possible. The possibility distribution π is normalized if $\exists\ \omega_i \in \Omega\ s.t,\ \pi(\omega_i) = 1$. Given a normalized possibility distribution π, we can describe the uncertainty about the occurrence of an event $A \subseteq \Omega$ via a possibility measure $\Pi(A) = \sup_{\omega_i \in A} \pi(\omega_i)$ and its dual necessity measure $N(A) = 1 - \Pi(\bar{A}) = 1 - \sup_{\omega_i \notin A} \pi(\omega_i)$. Measure $\Pi(A)$ evaluates to which extend A is *consistent* with the knowledge represented by π while $N(A)$ evaluates at which level A is certainly implied by the π. Conditioning in possibility theory is defined from the Bayesian-like equation $\Pi(A \cap B) = \Pi(A|B) \otimes \Pi(B)$, where $\otimes$ stands for the product in a quantitative setting (numerical) or for $\min$ in a qualitative setting (ordinal).

Possibilistic networks [1, 2] are defined as counterparts of Bayesian networks [12] in the context of possibility theory. They share the same basic components, namely:
(i) a *graphical component* which is a DAG (Directed Acyclic Graph) $\mathcal{G}= (V, E)$ where V is a set of nodes representing variables and E a set of edges encoding conditional (in)dependencies between them.
(ii) a *numerical component* associating a local normalized conditional possibility distribution to each variable $V_i \in V$ in the context of its parents (denoted by $pa(V_i)$). The two definitions of possibilistic conditioning lead to two variants of possibilistic networks: in the numerical context, we get product-based networks, while in the ordinal context, we get min-based networks (also known as qualitative possibilistic networks). Given a

possibilistic network , we can compute its encoded joint possibility distribution using the following chain rule: $\pi(V_1, \ldots, V_N) = \otimes_{i=1..N} \;\; \Pi(V_i \mid pa(V_i))$ where $\otimes$ is either the min or the *product* operator $*$ depending on the semantic underlying it.

3 Modeling preferences with a possibilistic network

In this section, we introduce a new approach, based on product-based possibilistic networks, for representing preferences. We use the product-based conditioning to avoid the drowning problem of the minimum operator and then increasing the discriminating power. In this approach, possibility degrees may remain symbolic but stands for numbers. As we shall see, the representation is particularly faithful to the user's preferences.

The ordering between interpretations obtained from this compact representation fully agrees with the inclusion ordering associated with the violation of preference statements. In the sense that if an interpretation ω_i violates all the preferences violated by another interpretation ω_j plus some other(s), then ω_i is strictly preferred to ω_j. Moreover, the relative importance of preferences can be easily taken into account when available. To illustrate the idea of representing preferences by means of possibilistic networks, we shall use the following example inspired from the CP-net literature [5].

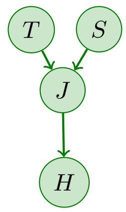
Example 1 *Let us consider a simple example about a night dressing with 4 variables standing for shirt (S), trousers (T), jacket (J) and shoes (H) s.t* $D(S) = \{black(s), red(\neg s)\}$, $D(T) = \{black(t), red(\neg t)\}$, $D(J) = \{red(j), white(\neg j)\}$ *and* $D(H) = \{white(h), black(\neg h)\}$. *The preference conditional set is:*

The user prefers to wear a black shirt to a red one.
He prefers to wear black trousers to red ones.
If he wears a black shirt and black trousers, he prefers to wear a red jacket to a white one.
If he wears a black shirt and red trousers, he prefers to wear a white jacket.
If he wears a red shirt and black trousers, he prefers to wear a red jacket.
If he wears a red shirt and red trousers, he prefers to wear a white jacket.
If he wears a red jacket, he prefers to wear white shoes to black ones.
If he wears a white jacket, he prefers to wear black shoes.

The universe of discourse associated to this example is:
$\Omega = \{\omega_1 = tjsh, \omega_2 = tjs\neg h,\ \omega_3 = tj\neg sh, \omega_4 = tj\neg s\neg h, \omega_5 = t\neg jsh, \omega_6 = t\neg js\neg h, \omega_7 = t\neg j\neg sh, \omega_8 = t\neg j\neg s\neg h, \omega_9 = \neg tjsh, \omega_{10} = \neg tjs\neg h, \omega_{11} = \neg tj\neg sh, \omega_{12} = \neg tj\neg s\neg h, \omega_{13} = \neg t\neg jsh, \omega_{14} = \neg t\neg js\neg h, \omega_{15} = \neg t\neg j\neg sh, \omega_{16} = \neg t\neg j\neg s\neg h\}$.

The preference description is assumed to be given under the form of conditional statements of the form $c : a \succ \neg a$ where c stands for the specification of a context in terms of Boolean variable(s) and a is a Boolean variable. Unconditional preferences correspond to the case where c is the tautology $\top$. The graphical structure of the network is then directly determined from this description (as in the CP-net case). Namely each variable corresponds to a node and conditional preferences are expressed by means of edges. The possibilistic preference table (ΠP-table for short) associated to a node is defined in the following way. To each preference of the form $c : a \succ \neg a$, pertaining to a variable A whose domain is $\{a, \neg a\}$, is associated the conditional possibility distribution $\Pi(a|c) = 1$ and $\Pi(\neg a|c) = \alpha$ where α is a symbolic weight such that $\alpha < 1$. We write $\Pi(\cdot|\top) = \Pi(\cdot)$.

$\Pi(t)$	$\Pi(\neg t)$
1	α



$\Pi(s)$	$\Pi(\neg s)$
1	β

$\Pi(.\|.)$	ts	$t\neg s$	$\neg ts$	$\neg t\neg s$
j	1	δ_2	1	δ_4
$\neg j$	δ_1	1	δ_3	1

$\Pi(.\|.)$	j	$\neg j$
h	1	ϵ_2
$\neg h$	ϵ_1	1

Fig. 1. A possibilistic network

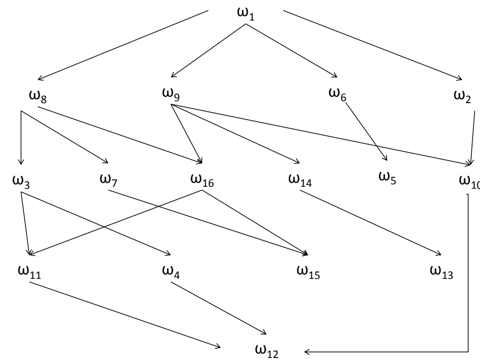


Fig. 2. The Inclusion-based ordering

Figure 1 gives the possibilistic graph associated to the Example 1. For instance, the corresponding conditional possibility distribution of the variable H is $\Pi(h|j) = 1$ and $\Pi(\neg h|j) = \epsilon_1$, $\Pi(\neg h|\neg j) = 1$ and $\Pi(h|\neg j) = \epsilon_2$. Thanks to conditional independence relations as exhibited by the graph, and using the product-based chain rule, we have: $\pi(TSJH) = \Pi(H|J) * \Pi(J|TS) * \Pi(T) * \Pi(S)$.

We are then in position to compute the symbolic possibility degree expressing the satisfaction level of any interpretation. For instance, $\pi(\omega_4) = \Pi(\neg h|j) * \Pi(j|t\neg s) * \Pi(t) * \Pi(\neg s) = \epsilon_1\delta_2\beta$. Similarly, $\pi(\omega_3) = \Pi(h|j) * \Pi(j|t\neg s) * \Pi(t) * \Pi(\neg s) = \delta_2\beta$. Then, based on the fact that $\forall\ \alpha,\ \alpha < 1$, and $\forall\alpha,\ \beta,\ \alpha * \beta < \min(\alpha, \beta)$, we can define a *partial order* $\succ_\Pi$ between interpretations under the form of a possibility distribution. In fact, given two interpretations $\omega_i,\ \omega_j \in \Omega$, $\omega_i \succ_\Pi \omega_j$ iff $\pi(\omega_i) > \pi(\omega_j)$. Thus, for instance, $\omega_3 \succ_\Pi \omega_4$. Besides, $\pi(\omega_6)= \delta_1$ and $\pi(\omega_{14}) = \alpha\delta_3$, thereby ω_6 and ω_{14} remain incomparable. However, if we further assume $\alpha < \delta_1$ expressing that the unconditional preference associated with a node T is more important than the preference $ts : j \succ \neg j$, we become in position to establish that $\omega_6 \succ_\Pi \omega_{14}$. Therefore, the approach leaves the freedom of specifying the *relative importance* of preferences.

Assume that for each node, i.e. each variable $V_i \in V$, two *distinct* symbolic weights are used, one for the context where the preferences associated with *each* parent nodes are satisfied, one *smaller* for all the other contexts. For instance, the symbolic weights of the variable J become $\delta_1 > \delta_2 = \delta_3 = \delta_4$ and those of the variable H become $\epsilon_1 > \epsilon_2$. The partial order induced from the possibilistic network (without adding other constraints between symbolic weights) is then faithful to the inclusion order associated to the violated constraints. It is, in fact, exactly the same ordering. This is due to the non comparability between some symbolic weights (following from the use of product). Figure 2 shows the inclusion-based order induced by the possibilistic graph with these additional assumptions. We should mention that the approach presented here can be extended to handle multivalued variables and cyclic preferences.

4 Comparison with CP-nets and concluding remarks

CP-nets [5] are based on the ceteris paribus principle. As can be seen on the previous example (where ω_6 and ω_{14} are incomparable, while $\top : t \succ \neg t$), possibilistic networks do not obey that latter principle. The order induced by the CP-net is a refinement of the possibilistic order $\succ_{\Pi}$, if no constraints about the relative importance of preferences are added. CP-nets are, in some sense, too *bold* and too *cautious*. Too bold since, as a result of the systematic application of the ceteris paribus principle, some priority is given to preferences associated to parent nodes which cannot be questioned and modified, as already said. Too cautious since they usually lead to a partial order while a complete preorder may be more useful in practice. The basic ordering associated to a possibilistic network is just the inclusion-based ordering, which can then be completed by adding relative importance constraints. In particular, a complete ordering of the symbolic weights leads to a complete preordering of the interpretations. It may be that CP-net orderings also respect the inclusion-based order, although it has apparently never been investigated.

Example 2 *Figures 3 and 4 show, respectively, the order induced by the CP-net and possibilistic network of Figure 1. Here we assume $\alpha = \beta < \delta_1 < \delta_2 = \delta_3 = \delta_4 < \epsilon_1 < \epsilon_2$. For instance, let us consider the interpretations ω_7 and ω_{16}. In contrast to the possibilistic network, which gives a total preorder, the CP-net considers these two interpretations as incomparable. We notice that both interpretations violate two preferences: associated to a parent and to a grandchild for ω_7, and to two parents preferences for ω_{16}. As expected, ω_7 is preferred to ω_{16} in the possibilistic network as their possibility degrees are respectively $\pi(\omega_7) = \beta\epsilon_2$ and $\pi(\omega_{16}) = \alpha\beta$.*

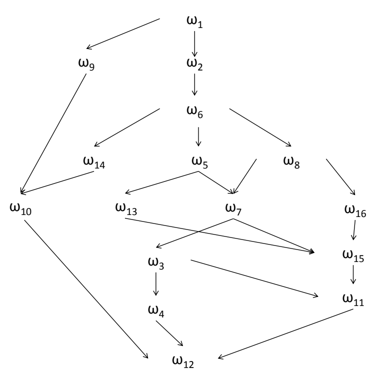


Fig. 3. The order induced by the CP-net

ω_1
ω_2
ω_6
$\omega_5, \omega_8, \omega_9$
ω_7, ω_{10}
ω_3, ω_{14}
$\omega_4, \omega_{13}, \omega_{16}$
ω_{15}
ω_{11}
ω_{12}

Fig. 4. The order induced by the possibilistic network

Moreover, CP-nets are sometimes unable to represent some user preferences.

Example 3 *Let us consider two binary variables A and B standing respectively for "vacations" and "good weather". Suppose that we have the following preference ordering: $ab \succ \neg a\neg b \succ a\neg b \succ \neg ab$. We observe that this complete preorder cannot be represented by a CP-net, while the possibilistic network can display it. In fact, given two variables we can define two possible structures: either A depends on B or inversely, both of them are unable to capture this order in the CP-net setting. However, such preferences can be represented by a joint possibility distribution such that: $\pi(ab) > \pi(\neg a\neg b) > \pi(a\neg b) > \pi(\neg ab)$. Since any joint possibility distribution can* *be decomposed into conditional possibility distributions as shown by the possibilistic chain rule, any complete preorder can be represented by a possibilistic net. Here, we have* $\top : a \succ \neg a$, $a : b \succ \neg b$ *and* $\neg a : \neg b \succ b$. *It corresponds to a network with two nodes with their corresponding conditional possibility distributions:* $\Pi(a) = 1$, $\Pi(\neg a) = \alpha$, $\Pi(b|a) = 1$, $\Pi(b|\neg a) = \gamma$, $\Pi(\neg b|a) = \beta$ *and* $\Pi(\neg b|\neg a) = 1$. *This yields* $\pi(ab) = 1 > \pi(\neg a \neg b) = \alpha > \pi(a \neg b) = \beta > \pi(\neg ab) = \alpha\gamma$ *taking* $\alpha > \beta$ *and* $\beta = \gamma$.

Lastly, it is important to mention that one of the advantages of the possibilistic graph is its ability to be translated into a possibility logic base [2, 9, 10] that can be used for executing the preference queries. This short note has outlined a preliminary presentation of possibilistic networks as providing a convenient setting for acyclic preference representation. This setting remains close to the spirit of Bayesian networks since it relies on directed acyclic graphs, but is flexible enough, thanks to the introduction of symbolic weights, for capturing any ordering agreeing with the inclusion-based ordering. Further research is still needed for investigating their potential in greater detail.

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