Evaluation of qualitative possibilistic influence diagrams using strong junction trees

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***Abstract*—Possibilistic influence diagrams are decision graphical models in the possibilistic framework [1]. They present an alliance between decision theory, graph theory and possibilistic theory, in order to represent decision problems and define their optimal strategy through evaluation algorithms. In this paper we present a new approach to evaluate qualitative influence diagrams.**

***Index Terms*—Possibilistic influence diagrams, decision theory, qualitative utilities.**

I. Introduction

In order to provide an optimal strategy relative to a decision problem we should explicitly define the context of decisions to be taken and the environment to which they are associated. Within existing graphical models to represent decision problems which are: decision trees [2], influence diagrams [3] and valuation based systems [4], we are in particular interested to influence diagrams. These models were introduced for the first time by Howard and Matheson [5], [6] and they offer a compact and natural modeling of decision problems under uncertainty which allows an ease determination of the optimal strategy and the computation of its value.

Influence diagrams can be defined as an alternative to decision trees as well as an extension to Bayesian networks. Thus, these graphical models overcome the limits of decision trees to represent decision problems (exponential size) and inherit the main features of Bayesian networks and enrich them with decision components.

Initially, these frameworks were proposed in a probabilistic context. However, it was proved that probability theory is appropriate only when all numerical information are available. Therefore, it presents some limits in regards to representation of total ignorance and qualitative uncertainty. This has favored the emergence of new uncertainty theories such as possibility theory which is a non-classical framework to handle uncertainty, introduced by Zadeh [7] and developed by Dubois and Prade [8]. The development of this theory gave birth to several possibilistic graphical models such as *possibilistic influence diagrams* [1] which are the possibilistic counterpart of standard influence diagrams [3].

These models outline decision problems proposed in a possibilistic context and their evaluation provide the optimal strategy relative to this problem. Thus, the theoretical complexity of evaluation algorithms relative to (probabilistic or possibilistic) influence diagrams is equivalent to the complexity of inference in Bayesian networks which is stated as an NP-hard problem [9].

Solving this evaluation problem presents the subject of several research works. For example, we can mention [1], [3], [6], [10] but the majority of these works refer to the probabilistic framework and there are just few of them which are dedicated to evaluate possibilistic influence diagrams. Through this article we aim to present a new indirect method to evaluate possibilistic qualitative influence diagrams using strong junction trees [11], [12]. The remaining of this paper is organized as follows: In section II we will focus on the definition of possibilistic influence diagrams and their graphical and numerical components. Section III proposes our new approach to evaluate possibilistic influence diagrams quantified with consideration to the qualitative scale.

II. Possibilistic influence diagrams

In real world it is not always obvious to provide numerical distributions representing uncertainty relative to the decision problem's variables. Besides, there are some situations where we can be faced by problems with total ignorance. So, in these cases the use of probabilistic reasoning will be unsound. However, possibility theory presents a non classical framework appropriate to express uncertainty numerically using possibility degrees or qualitatively by defining a rank between the different states of a given variable. This qualitative representation seems further suitable for real problems because it's more leisure for experts to provide a preference order towards different strategies than estimating numerical values.

Possibilistic influence diagrams benefit from this efficiency of possibility theory to deal with uncertainty, as well as the structural simplicity of classical influence diagrams [3]. So, they can be defined as an assembly of graphical and numerical components, that share the same graphical component as classical (probabilistic) influence diagrams while the quantification is performed differently.

More precisely, the graphical component resumes the decision problems variables and emphasizes the dependency/independency relationships between them. These variables are represented by a set of nodes $\mathcal{N}$ composed of three subsets $\mathcal{C}$, $\mathcal{D}$ and $\mathcal{V}$ such that:

- ***Chance nodes*** $\mathcal{C} = \{\mathcal{C}_\infty, ..., \mathcal{C}_\backslash\}$ depicts a set of (n) random uncertain variables representing states of the world. The values of these variables are affected by each others according to the relations of dependency between them, but not affected by decisions to be made. They are usually drawn as circles.
- ***Decision nodes*** $\mathcal{D} = \{\mathcal{D}_\infty, ..., \mathcal{D}_\surd\}$ depicts a set of (p) decisions under control of the decision maker, and they are numbered according to the order in which they will be made. Schematically, they are represented by squares.
- ***Value nodes*** $\mathcal{V} = \{\mathcal{V}_\infty, ..., \mathcal{V}_\Pi\}$ depicts a set of (q) result variables which represent the objective sought to maximize. They express the utility of the problem. They are drawn as diamonds.

The dependencies and relationships between these nodes are defined by a set of arcs $\mathcal{A}$ such that each arc can be:

- ***Conditional arc***: Connecting two nodes with a chance or a value node as a target.
- ***Informational arc***: Connecting two nodes with a decision node as a target.

Given an influence diagram, we can define a temporal order such that:
For each decision $D_i \in \mathcal{D}$ a set of ordered decisions, there exists a set of chance variables $SC_i \in \mathcal{C} = \{\mathcal{SC}_\rangle ... \mathcal{SC}_\surd\}$ observed between decisions D_i and D_{i+1}. SC_0 (resp. SC_p) presents the set of chance variables observed before (resp. after) decision D_1 (resp. D_p).

The numerical component, is defined by a range of various values describing the quantification of the diagram. This quantification is performed by assigning possibility degrees $\Pi(C_i|Par(C_i))$ to chance nodes C and utility functions $U(Par(V_i))$ to value nodes V. Where $Par(N_i)$ present the set of parents of the node N_i. This quantification for chance nodes as well as value nodes can be assigned independently on a numerical or ordinal possibilistic scale.

Example 1: Let us consider figure 1 to represent the graphical component of a possibilistic influence diagram. The outlined structure is defined by $\mathcal{C}$=$\{A_1, A_2\}$, $\mathcal{D}$=$\{D_1, D_2\}$ and $\mathcal{V}$=$\{V\}$. Tables I, II and III resume the quantification of this structure.

A_1	D_1	$\Pi(A_1\|D_1)$
T	T	1
T	F	0.4
F	T	0.2
F	F	1

TABLE I
CONDITIONAL POSSIBILITY OF A_1

A_2	D_2	$\Pi(A_2\|D_2)$
T	T	0.3
T	F	1
F	T	1
F	F	0.4

TABLE II
CONDITIONAL POSSIBILITY OF A_2

D_1	D_2	A_2	$u(D_1, D_2, A_2)$
T	T	T	0.2
T	T	F	0.3
T	F	T	0.4
T	F	F	0.6
F	T	T	1
F	T	F	0
F	F	T	0.1
F	F	F	0.7

TABLE III
UTILITY FUNCTION OF (D_1, D_2, A_2)

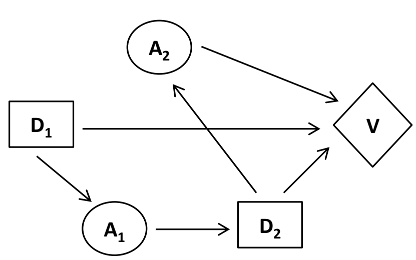

Fig. 1. Example of graphical component of a possibilistic influence diagram

According to the quantification of the possibilistic influence diagram, this latter can be a *product-based* one where both chance and value nodes are quantified according to a numerical possibilistic scale or a *min-based* one where both chance nodes and value nodes are quantified according to an ordinal possibilistic scale. In this work we will focus on qualitative possibilistic influence diagrams (*min-based* possibilistic influence diagrams) which are denoted by $\Pi\mathcal{ID}_{min}$.

III. NEW EVALUATION ALGORITHM OF POSSIBILISTIC INFLUENCE DIAGRAMS

Evaluating influence diagrams is the process adopted to determinate the optimal solution that solves an outlined decision problem. It refers to the inference task applied for the diagram by choosing the best decision alternatives in the aim to maximize the utility function and to obtain the optimal strategy. An optimal strategy can be defined as follows:

Definition 1: The optimal strategy is a sequence of decision rules or policies denoted by $\Delta = \{\delta_i, ..., \delta_p\}$ where δ_i is the best decision rule relative to D_i, providing the highest utility, with regards to a decision criterion $\mathcal{O}$.
Although the literature is rich of research works that treat the problem of evaluating influence diagrams, there are just few of them which are dedicated to evaluate possibilistic ones. We can mention in particular, Garcia et al. [1], [13] that propose two evaluation methods where the first one consider a direct evaluation of possibilistic influence diagrams and the second one allows their evaluation after transforming them into possibilistic decision trees. Besides, we can cite [14] where authors provide a direct evaluation adequate also to $\Pi\mathcal{ID}_{min}$ using order of magnitude expected utility (OMEU). Finally, we cite [15] which is a recent research that offers the possibility to evaluate possibilistic influence diagrams with all possibilistic decision criteria through two indirect methods based on transformation into possibilistic decision trees and possibilistic networks.

In probabilistic context, solving a decision problem is generally done on the basis of maximal expected utility (MEU). So, this decision criterion was appropriate to evaluate probabilistic influence diagrams. With the development of possibility theory, there was a need to find the possibilistic counterpart of this criterion that would be used to evaluate possibilistic influence diagrams. In this regard, several researches were devoted to study this problem, and they have given rise to panoply of decision criteria, which are defined with consideration of the used possibilistic scale.

Among different possibilistic decision criteria we are in particular interested to optimistic (U_{opt}) and pessimistic (U_{pes}) utilities [16], [17] which are based on the behavior of decision maker and present the counterpart of the expected utility in a purely qualitative context and they are computed respectively as follows:

$$U_{opt}(d) = \max_{x \in X} \; \min \; (\pi(x), u(d(x))) \tag{1}$$

$$U_{pes}(d) = \min_{x \in X} \; \max \; (1 - \pi(x), u(d(x))) \tag{2}$$

So, in the following, we will present a new approach to evaluate min-based possibilistic influence diagrams using either (U_{opt}) or (U_{pes}). Our method constitutes an adaptation of the probabilistic approach presented by Jensen [11] and it consists on performing the graphical manipulation to transform influence diagrams into strong junction trees then applying the appropriate inference algorithm to deduce the optimal strategy. The choice of this method is justified by the fact that junction tree algorithm is the only graphical model that allows inference in singly as well as multiply connected graphs.

A. Graphical transformation

The first step in our evaluation algorithm consists on transforming initial possibilistic influence diagrams into a secondary structure i.e. strong junction trees, denoted by ($\Pi\mathcal{SJT}$) and composed of a set of cliques $\mathcal{C}\updownarrow$, containing m nodes, where each two cliques are connected via a separator $\mathcal{S}$. The numerical component of this structure is defined by possibility and utility potentials.

Since the construction of the strong junction tree is totally independent of the numerical values, we use the same steps than those initially proposed for standard influence diagrams [11] i.e.

1) Remove informational arcs.
2) Moralization of the graph.
3) Remove utility nodes.
4) Triangulation of the moral graph.
5) Construction of the strong junction tree.

Example 2: Figure 2 represents the result of the graphical transformation performed on the initial diagram presented in figure 1. The possibilistic strong junction tree presented in this figure contains two clusters: $A_2D_1D_2$ (the root) and A_1D_1, separated with the separator D_1. The two clusters are constructed by eliminating variables from the initial diagram presented in figure 1 and the separator encompasses the common variables between these clusters.

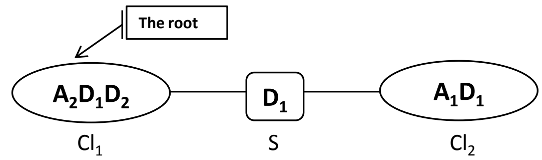

Fig. 2. The possibilistic strong junction tree corresponding to the initial influence diagram presented in figure 1

Once the construction of the possibilistic strong junction tree is performed, we carry on to the second step which is the *absorption* process consisting in a particular propagation, that allows the inference among clusters with marginalization operations. Thus, the evaluation problem is reduced to inference in possibilistic strong junction trees. This phase constitutes the core of our approach and is detailed in what follows:

B. Possibilistic absorption

After proceeding graphical transformation and to achieve our goal which amounts to the evaluation of the initial diagram, we have to perform the possibilistic absorption on the obtained structure i.e. the strong junction tree ($\Pi\mathcal{SJT}$). This structure presents the possibilistic qualitative counterpart of the strong junction tree presented by Jensen in [11] while we are interested only on $\Pi\mathcal{ID}_{min}$.

This absorption consists on assigning for each cluster Cl_i (resp. separator S_i) of the $\Pi\mathcal{SJT}$ two local potentials which are *possibility potential* and *utility potential*. The first one defines the possibility joint distribution associated to each cluster and separator denoted by Φ_{Cl_i}(resp. Φ_{S_i}) and the second one refers to the utility degree relative to each cluster and separator denoted by Ψ_{Cl_i} (resp. Ψ_{S_i}).
Besides, the absorption process is based on message passing mechanism which allows the updating of clusters and separators potentials and it's carried out in two phases: *Initialization* and *Global absorption*.

1) Initialization: The initialization of any $\Pi\mathcal{SJT}$ consists on quantifying its structure with initial possibility distributions and utility functions. The possibility potential for each cluster corresponds to its local joint distribution which is computed from the initial diagram as follows:

$$\Phi_{Cl_i} = \min_{j:1..m} \Pi(C_{ij} | Par(C_{ij})) \tag{3}$$

where C_{ij} is a chance node C_j in the cluster Cl_i.

Furthermore, for each cluster we assign an utility potential that depicts an ordinal satisfaction towards the different strategies for $\Pi\mathcal{SJT}$, such as:

$$\Psi_{Cl_i} = \min_{j:1..r} U_j(Par'(V_i)) \tag{4}$$

where $Par'(V_i) = Par(V_i)$ in Cl_i. $U_j(Par'(V_i))$ is the utility relative to each chance or decision node in Cl_i which was a direct parent of the value node V_i.

For clusters (resp. separators) without possibility potential, we assign the unity potential and for clusters (resp. separators) without utility potential we assign the null utility potential. Besides, if a chance node C_j exists in more than cluster,

	Cl_1	S	Cl_2
Φ	$TableII$	1	$TableI$
Ψ	$TableIII$	0	0

TABLE IV
INITIALIZATION OF THE OBTAINED $\Pi\mathcal{SJT}$

its possibility degree $\Pi(C_j|Par(C_j))$ should be assigned to one and only one clique Cl_i, which is the closest to the leaf. Algorithm 1 outlines the initialization process corresponding to $\Pi\mathcal{SJT}$.

Algorithm 1: Initialization of strong possibilistic junction trees

Data: Initial distributions and utilities, constructed $\Pi\mathcal{SJT}$
Result: Initialized $\Pi\mathcal{SJT}$
begin
  foreach $Cl_i \in \mathcal{CL}$ **do**
    $\phi^I_{Cl_i} \leftarrow 1$; $\psi^I_{Cl_i} \leftarrow 0$;
  foreach $S_i \in \mathcal{S}$ **do**
    $\phi^I_{S_i} \leftarrow 1$; $\psi^I_{S_i} \leftarrow 0$;
  foreach $X \in \mathcal{C}$ **do**
    - Choose $Cl_i \in \mathcal{CL}$ (a leaf or a closer cluster) containing $\{X\} \cup Par(X)$:
    $\phi^I_{Cl_i} \leftarrow \min(\phi^I_{Cl_i}, \Pi(X|Par(X)))$;
    foreach $X \in Par(V_i)$ **do**
      - Choose a cluster $Cl_i \in \mathcal{CL}$ (a leaf or a closer cluster) containing $\{X\} \cup Par(X)$:
      $\psi^I_{Cl_i} \leftarrow \min(\psi^I_{C_i}, U(X|Par(X)))$;
end

Example 3: The initialization process concerns the numerical component relative to the obtained strong junction tree. For our example, the quantification of the structure presented by figure 2 can be resumed as follows:

2) Global absorption: The global absorption or propagation in $\Pi\mathcal{SJT}$ is similar to the *collect process* in classical junction trees propagation algorithms. The absorption is performed from leafs to the root via message passing mechanism, where exchanged messages allow updating of clusters and separators potentials. This procedure should be performed after the choice of the decision criterion that would be used to evaluate the initial diagram and it's based on the marginalization operation. The latter presents the counterpart of variable elimination in the graphical manipulation.

Therefore, in our approach for the computation of the optimal strategy we have to proceed by eliminating variables included in clusters starting from leafs to the root of the obtained $\Pi\mathcal{SJT}$, according to the reverse of the temporal order of the initial possibilistic influence diagram. As mentioned above, this evaluation can be performed for $\Pi\mathcal{ID}_{min}$ using optimistic (U_{opt}) or pessimistic (U_{pes}) utility regarding to the decision maker's attitude.

Proposition 1: Let $\vec{d} = \{d_1, ..., d_p\}$ and $\vec{c} = \{c_1, ..., c_n\}$ be respectively a set of alternatives relative to decision variables $\mathcal{D}$ and chance variables $\mathcal{C}$ that constitute the set of all variables $\mathcal{W}$ ($\mathcal{W} = \mathcal{D} \cup \mathcal{C}$). Let f($\vec{c}$,$\vec{d}$) defines the function relative to possibility and utility potentials relative to $\mathcal{W}$, such as : $f(\vec{c}, \vec{d}) = \min(\min_i \Pi_i(\vec{c}, \vec{d}), \min_i U_i(\vec{c}, \vec{d}))$. For every removed variable W_i (i.e. chance or decision node), after marginalizing on it, the optimal strategy relative to this decision problem, can be defined as follows:
Using U_{opt}: $Arg\max_{\vec{d}} \max_{\vec{c}} f'(\vec{c}, \vec{d})$

$$= \begin{cases} \sum^* Arg\max_{\vec{d}} \max_{\vec{c''}} f((\vec{c'} \cdot \vec{c''}), \vec{d}) & \text{if } W_i \in \text{C} \\ \sum^* Arg\max_{\vec{d''}} \max_{\vec{c}} f(\vec{c}, (\vec{d'} \cdot \vec{d''})) & \text{if } W_i \in \text{D} \end{cases} \quad (5)$$

Using U_{pes}: $Arg\max_{\vec{d}} \min_{\vec{c''}} f((\vec{c'} \cdot \vec{c''}), \vec{d})$

$$= \begin{cases} \sum^* Arg\max_{\vec{d}} \min_{\vec{c''}} f((\vec{c'} \cdot \vec{c''}), \vec{d}) & \text{if } W_i \in \text{C} \\ \sum^* Arg\max_{\vec{d''}} \min_{\vec{c}} f(\vec{c}, (\vec{d'} \cdot \vec{d''})) & \text{if } W_i \in \text{D} \end{cases} \quad (6)$$

where $\sum^*$ refers to the marginalization operator, $\vec{c'}$ is the set of eliminated alternatives and $\vec{c''} = \vec{c} \backslash \vec{c'}$. $\vec{d'}$ is the set of eliminated alternatives and $\vec{d''} = \vec{d} \backslash \vec{d'}$.

Algorithm 2 allows the global absorption for $\Pi\mathcal{SJT}$ with consideration to these criteria. The computation of equations mentioned in this algorithm are computed as follows:

1) Conditional potential of separator S_k:

$$\Phi_{S_k} = \sum_{Cl_i \backslash S_k}^{*} \Phi_{Cl_i} \quad (7)$$

2) Utility potential of separator S_k:

$$\Psi_{S_k} = \ominus_{Cl_i \backslash S_k} \odot (\Psi_{Cl_i}, F(\Phi_{Cl_i})) \quad (8)$$

where F(Φ_{Cl_i}) corresponds to Φ_{Cl_i} (resp. $1 - \Phi_{Cl_i}$) using U_{opt} (resp. U_{pes}), $\ominus$ is the max (resp. the min) operator using U_{opt} (resp. U_{pes}) and $\odot$ is the min (resp. the max) operator using U_{opt} (resp. U_{pes}).

3) Conditional potential of cluster Cl_j:

$$\Phi^{new}_{Cl_j} = \min(\Phi_{Cl_j}, \Phi_{S_k}) \quad (9)$$

4) Utility potential of cluster Cl_j:

$$\Psi^{new}_{Cl_j} = \ominus_{Cl_i \backslash S_k} \odot (\Psi_{S_k}, F(\Phi_{Cl_j})) \quad (10)$$

The optimal strategy δ^* is composed of a set of policies that maximize each decision. The optimal policy δ^*_i for each decision node D_i can be inferred from the root or the closest cluster containing this decision node, such that:

$$\delta^*_i = \arg\max_{D_i} \Psi_{Cl_j} \quad (11)$$

Thus, the value of the optimal strategy for $\Pi\mathcal{SJT}$ is determined from the root after message passing and its value is the value that maximizes the utility potential of this cluster.

Example 4: For the calculation of the optimal strategy using U_{opt} and U_{pes}, we have to compute the possibility and utility potentials for Φ_S and Φ_{Cl_1} using Equations 7, 9, 8 and 10. These calculations are detailed in tables V, VIII, VI, VII. After potentials updating, we obtain δ^*={(D_2=F),(D_1=T/F)}=0.4 (resp. δ^*={(D_2=F),(D_1=T)}=0.4) using U_{opt} (resp. U_{pes}).

Algorithm 2: Global propagation

Data: Initialized $\Pi\mathcal{SJT}$
Result: Updated $\Pi\mathcal{SJT}$
begin
- Let *Branch* be an ordered vector containing clusters and their separators from leaf to the root. % *Branch*[i] is a cluster or separator in *Branch*.
- Let L defines the length of each cluster (i.e. number of clusters and separators contained in *Branch*).
- Let *Decision-criterion* be the chosen criterion to evaluate the initial diagram which can be U_{opt} or U_{pes}.
foreach *Branch in* $\Pi\mathcal{SJT}$ **do**
  for $i \leftarrow$ *1 to L-1* **do**
    $Cl_i \leftarrow$ Branch [i];
    $S_k \leftarrow$ Branch [i+1];
    $Cl_j \leftarrow$ Branch [i+2];
    - Post a message from Cl_i to S_k using (7) and (8);
    - Post a message from S_k to Cl_j using (9) and (10);
end

D_2	A_2	$\Pi(A_2\|D_2)$	Φ_S
T	T	0.3	$\max(1, 0.3) = 1$
T	F	1	
F	T	1	$\max(1, 0.4) = 1$
F	F	0.4	

TABLE V
POSSIBILITY POTENTIAL OF S USING U_{opt} OR U_{pes}

D_1	D_2	A_2	Ψ_{Cl_2}	Φ_{Cl_2}	$1-\Phi_{Cl_2}$	$\Psi_{Cl_2\backslash A_2}$ (U_{opt})	$\Psi_{Cl_2\backslash A_2}$ (U_{pes})	Ψ_S (U_{opt})	Ψ_S (U_{pes})
T	T	T	0.2	0.3	0.7	0.3	0.3	0.4	0.4
T	T	F	0.3	1	0				
T	F	T	0.4	1	0	0.4	0.4		
T	F	F	0.6	0.4	0.6				
F	T	T	1	0.3	0.7	0.3	0	0.4	0.1
F	T	F	0	1	0				
F	F	T	0.1	1	0	0.4	0.1		
F	F	F	0.7	0.4	0.6				

TABLE VI
UTILITY POTENTIAL OF S

A_1	D_1	Φ_{Cl_1}	$1-\Phi_{Cl_1}$	Ψ_S (U_{opt})	Ψ_S (U_{pes})	Ψ_{Cl_1} (U_{opt})	Ψ_{Cl_1} (U_{pes})
T	T	1	0	0.4	0.4	0.4	0.4
T	F	0.4	0.6	0.4	0.4		
F	T	0.2	0.8	0.4	0.1	0.4	0.1
F	F	1	0	0.4	0.1		

TABLE VII
UTILITY POTENTIAL OF Cl_1

A_1	D_1	Φ_{Cl_1}	Φ_S	$\Phi_{Cl_1}^{new}$
T	T	1	1	$\min(1, 1) = 1$
T	F	0.4	1	$\min(0.4, 1) = 0.4$
F	T	0.2	1	$\min(0.2, 1) = 0.2$
F	F	1	1	$\min(1, 1) = 1$

TABLE VIII
POSSIBILITY POTENTIAL OF Cl_1 USING U_{opt} OR U_{pes}

IV. CONCLUSION

In this work we have proposed a new indirect method to evaluate possibilistic qualitative influence diagrams which can be applied for singly and sequential decision problems as well as the consideration of influence diagrams with one or many value nodes. The basics of our idea were inspired from Jensen's method [11], which is based on transformation of influence diagrams into strong junction trees.

We chosen this approach, which profits from the properties of junction trees, because it differs from the methods presented by Garcia et al. in [1], [13] by its ability to evaluate non-regular influence diagrams, when their structure contains cycles. So, we adapted this method to benefit from its advantage to evaluate influence diagrams, which are quantified in a possibilistic scale. Besides, we can consider that our approach presents a simplification of the general decision making framework defined by Pralet et al. in [18]. In fact, we use the same aggregation operators for utilities and possibilities.

As future work, we can consider direct improvements of our proposal to make it suitable for all types of possibilistic influence diagrams with several decision criteria, in order to provide a complete complexity study of decision making using possibilistic influence diagrams. Moreover, it will be interesting to develop the possibilistic counterpart of particular classes of influence diagrams such as unconstrained influence diagrams (UIDs) and sequential influence diagrams (SIDs) [19] which can cope with asymmetric decision problems.

APPENDIX A
PROOF OF ELIMINATING VARIABLE USING U_{opt}

Let $\vec{d} = \{d_1, ..., d_p\}$ and $\vec{c} = \{c_1, ..., c_n\}$ be respectively a set of alternatives relative to decision nodes D and chance nodes C that outline the decision problem. For any decision vector $\vec{d}$ the utility based on U_{opt} is defined by: $U_{opt}(\vec{d}) = \max_{\vec{c}} \min(\Pi(\vec{c}, \vec{d}), U(\vec{c}, \vec{d}))$ where $\Pi(\vec{c}, \vec{d}) = \min_i \Pi_i(\vec{c}, \vec{d})$ and $U(\vec{c}, \vec{d}) = \min_i U_i(\vec{c}, \vec{d})$.

Our objective is to find the best decision alternatives that maximize U_{opt}, which amounts to determine (i) Arg $\max_{\vec{d}} \max_{\vec{c}} \min(\min_i \Pi_i(\vec{c}, \vec{d}), \min_i U_i(\vec{c}, \vec{d}))$. Let f $(\vec{c}, \vec{d})$=min($\min_i \Pi_i(\vec{c}, \vec{d}), \min_i U_i(\vec{c}, \vec{d})$). So, we can rewrite (i) by: (ii) $Arg\max_{\vec{d}} \max_{\vec{c}} f(\vec{c}, \vec{d})$.

In order to solve (ii), our strategy consists on eliminating variables, one by one according to the reverse of the temporal order of $\Pi\mathcal{ID}$, as mentioned in section III. This variable elimination can be expressed mathematically by the marginalization operation. Let $f^{'}(\vec{c}, \vec{d})$ defines the function $f(\vec{c}, \vec{d})$ after eliminating a variable. This variable can be a chance variable or a decision variable, and its elimination denotes the elimination of a set of alternatives relative to the eliminated variable. If the eliminated variable is a chance variable C_i, then we have to show that $f'(\vec{c}, \vec{d}) = \max_{C_i} f(\vec{c'} \cdot \vec{c''}, \vec{d})$, where $\vec{c'}$ is the set of eliminated alternatives relative to C_i and $\vec{c''} = \vec{c} \backslash \vec{c'}$. If the eliminated variable is a decision variable D_i then we have to show that $f'(\vec{c}, \vec{d})$ = $\max_{D_i} f(\vec{c}, \vec{d'} \cdot \vec{d''})$, where $\vec{d'}$ is the set of eliminated alternatives relative to D_i and $\vec{d''} = \vec{d} \backslash \vec{d'}$

To facilitate the calculation, we take advantage of the fact that the operator min respects the associativity. So, we can gather the set of all possibilities $\Pi(\vec{c}, \vec{d})$ and the set of all utilities U $(\vec{c}, \vec{d})$ to a set of potentials $\Theta(\vec{c}, \vec{d}) = \{\Theta_1, ..., \Theta_n\}$ and we replace $f'(\vec{x}, \vec{d})$ by $f'(\vec{y})$ where $\vec{y}$ is the association of a decision and a chance vectors. Suppose that we will eliminate the variable V, so we obtain $\vec{y''} = y \backslash \vec{y'}$, where $\vec{y'}$ is the set of eliminated alternatives. Thus, we have to prove that $f'(\vec{y}) = \max_{y} f(\vec{y'} \cdot \vec{y''})$. Then, (ii) can be defined as follows: $Arg\max_{\vec{d}} \max_{\vec{c}} \min_{\Theta_i \in \Theta} \Theta_i(\vec{c}, \vec{d})$ where $\Theta_i = \Pi_i \cup U_i$ and var Θ_i defines the set of variables that their potentials are expressed by Θ_i.

When applying the decision criterion U_{opt}, the marginalization is done respectively for chance nodes and decision nodes with the max operator. So we have:

$$\text{Arg} \max_{\vec{d}} \max_{\vec{c}} \min_{\Theta_i \in \Theta} \Theta_i(\vec{x}, \vec{d})$$
$$= \max_{V} \min_{\Theta_i \in \Theta} \Theta_i(\vec{y})$$
$$= \max_{V} \min(\min_{V \notin var(\Theta_i)} \Theta_i, \min_{V \in var(\Theta_i)} \Theta_i)$$
$$= \min(\min_{V \notin var(\Theta_i)} \Theta_i, \max_{V} \min_{V \in var(\Theta_i)} \Theta_i)$$

This allows us to conclude that the determination of the optimal strategy using U_{opt} can be done by elimination of variables according to the reverse of the temporal order of the initial diagram.

Appendix B
Proof of eliminating variable using U_{pes}

Let $\vec{d} = \{d_1, ..., d_p\}$ and $\vec{c} = \{c_1, ..., c_n\}$ be respectively a set of alternatives relative to decision nodes D and chance nodes C that outline the decision problem. For any decision vector $\vec{d}$ the utility based on U_{pes} is defined by: $U_{pes}(\vec{d}) = \min_{\vec{c}} \max(1 - \Pi(\vec{c}, \vec{d}); U(\vec{c}, \vec{d}))$ where $\Pi(\vec{c}, \vec{d}) = \min_{i} \Pi_i(\vec{c}, \vec{d})$ and $U(\vec{c}, \vec{d}) = \min_{i} U_i(\vec{c}, \vec{d})$

Our objective is to find the best decision alternatives that maximize U_{pes}, which amounts to determine (i) $Arg\max_{\vec{d}} \min_{\vec{c}} \max(1 - \min_{i} \Pi_i(\vec{c}, \vec{d}), \min_{i} U_i(\vec{c}, \vec{d}))$.

Let f $(\vec{c}, \vec{d})$=$\max(1 - \min_{i} \Pi_i(\vec{c}, \vec{d}), \min_{i} U_i(\vec{c}, \vec{d}))$. So, we can rewrite the (i) by (ii) $Arg\max_{\vec{d}} \min_{\vec{c}} f(\vec{c}, \vec{d})$.

In order to solve (ii), our strategy consists on eliminating variables, one by one according to the reverse of the temporal order of $\Pi\mathcal{ID}$, as mentioned in section III. This variable elimination can be expressed mathematically by the marginalization operation. Let $f'(\vec{c}, \vec{d})$ defines the function $f(\vec{c}, \vec{d})$ after eliminating a variable. This variable can be a chance variable or a decision variable, and its elimination denotes the elimination of a set of alternatives relative to the eliminated variable. If the eliminated variable is a chance variable C_i, then we have to show that $f'(\vec{c}, \vec{d}) = \max_{C_i} f(\vec{c'} \cdot \vec{c''}, \vec{d})$, where $\vec{c'}$ is the set of eliminated alternatives relative to C_i and $\vec{c''} = \vec{c} \backslash \vec{c'}$. If the eliminated variable is a decision variable D_i then we have to show that $f'(\vec{c}, \vec{d})$ = $\max_{D_i} f(\vec{c}, \vec{d'} \cdot \vec{d''})$, where $\vec{d'}$ is the set of eliminated alternatives relative to D_i and $\vec{d''} = \vec{d} \backslash \vec{d'}$.

For our approach using U_{pes}, the marginalization is done respectively for chance nodes and decision nodes with the min operator.

Let us consider V the variable to be eliminated, which can be a decision or a chance variable. So, we get:

$$\min_{V} \max(1 - \Pi; U)$$
$$= \min_{V} \max(\min(1 - \min_{V \in var(i)} \Pi_i, 1 - \min_{V \notin var(i)} \Pi_i); \min(\min_{V \in var(i)} U_i, \min_{V \notin var(i)} U_i))$$
$$= \max(\min_{V} \min(1 - \min_{V \in var(i)} \Pi_i, 1 - \min \prod_{V \notin var(i)_i}); \min_{V} \min(\min_{V \in var(i)} U_i, \min_{V \notin var(i)} U_i))$$
$$= \max(\min(\min_{V} 1 - \min_{V \in var(i)} \Pi_i, 1 - \min \prod_{V \notin var(i)_i}); \max \min(\min_{V} \min_{V \in var(i)} U_i, \min_{V \notin var(i)} U_i))$$
$$= \min(\max(\min_{V} 1 - \min_{V \in var(i)} \Pi_i, 1 - \min \prod_{V \notin var(i)_i}); \max \min(\min_{V} \min_{V \in var(i)} U_i, \min_{V \notin var(i)} U_i))$$

This allows us to conclude that the determination of the optimal strategy using U_{pes} can be done by variable elimination according to the reverse of temporal order of the initial diagram.

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