A generic algorithm for traversing a Directed Acyclic Graph: a modeling and solving approach for the Home Health Care Scheduling Problem

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Abstract

Recently, Hierarchical Optimization Models (HOM) has been widely used to model combinatorial optimization problems (COP) in many fields such as healthcare, supply chain, transport, economic, etc... Their efficiency as a modeling approach is explained by their ability of relaxing the complexity of the considered COP. The main problem will be decomposed hierarchically into a set of interconnected sub-problems using multiple decomposition strategies. However, solving the hierarchical optimization model presents a great challenge and difficult task to get an optimal solution for the main problem. In this context, we propose a generic algorithm to efficiently solve the hierarchical optimization model. Moreover, the developed algorithm will be able to solve all kind of hierarchical optimization models using any decomposition strategy. To validate the proposed algorithm, the HCSP will be solved using the proposed modeling and solving approach.
Keywords: Hierarchical Optimization Framework, Graph Traversal Algorithm, Directed Acyclic Graph, Based Objective Decomposition Strategy, Semantic Decomposition Strategy, Constraint Relaxation Decomposition Strategy, Data Partitioning Strategy.

1 Introduction

The research paper can be situated in the field of health care study from an analytical point of view. Particularly, it focuses on home health care services and how their operational schedules can be enhanced using Operations Research (OR) models and solving algorithms. Besides, the HCSP faces some conflicting goals, constraints and preferences. In such conditions, a predefined modeling and solving strategy will significantly relax the problem and by result facilitate to the decision maker to find a satisfying solution approach that takes into account the humanistic judgments and the conflicting nature of goals, constraints and preferences. In this context, we present a hierarchical optimization framework (HOF) as a decision support modeling tool to supervise the choice of the best modeling approach based on set of problem' properties. The proposed HOF is composed by four decomposition strategies.

The output of the hierarchical optimization framework is a Directed Acyclic Graph (DAG). The DAG is composed by a set of interconnected sub-problems presented by a set of nodes and related by a set of edges. During the solving process, partial solutions and solving contexts should be communicated to other sub-problems following the decomposition rules to find the final solution. In this context, two relationships types may exist between sub-problems which are: (i) a decomposition relationship or (ii) a dependency relationship.

In the second part of the paper, we aim to develop a graph traverse algorithm that will represent a tool for visiting all sub-problems in a hierarchical optimization model. The proposed generic algorithm will respect the structure of hierarchical model. The DAG will be traversed taking into consideration the different dependencies between the components of the main decomposed problem.

In the third part, we present our approach to model and solve the HCSP. The HCSP will be modeled as a hierarchical optimization problem and will be traversed using the generic algorithm and its components will be solved.

2 The HOP, the DAG and the HCSP model descriptions

2.1. HOP Description

To model complex HOP, we draw the following framework which is composed by four strategies (see Fig. 1).

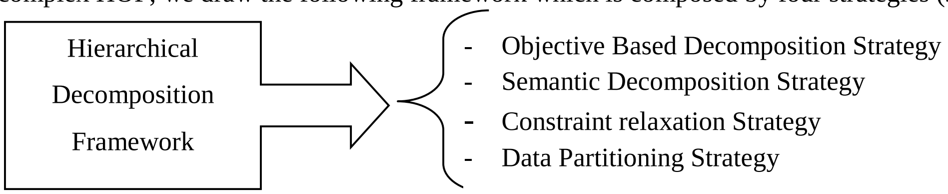

Fig. 1. Hierarchical decomposition strategies

2.2. DAG description

The output of the hierarchical modeling approach presented previously is a Directed Acyclic Graph (DAG). Each node of the graph is a sub-problem to solve. Leaves (terminal nodes) represent trivial problem or ready solved problems. Intermediate nodes are sub-problems being decomposed into other sub-problems in lower levels. Additionally, the descendants of a non-terminal node can be dependent or independent. Therefore, the obtained DAG has two types of edges:

- Decomposition edges showing a link between a master problem and its components (sub-problems).
- Dependency edges between sub-problems of the same level and generated from the same master problem.

2.3. The HCSP model

The Home Care Scheduling Problem can be viewed as a particular combination and ordering of other optimization problems. The solution of the initial problem can be rebuilt by combining solutions of its sub-problems. There, depending on how the HCSP is defined, one of the fourth decomposition strategies can be applied to hierarchically model the problem and derive its sub-problems. In our case, we define the HCSP as a combination of essentially three components which are: (i) the grouping sub-problem, (ii) the assignment sub-problem and (iii) the routing sub-problem. Generally, such sub-problems involve a set of decisions that have to be token in a particular sequence (order) due to the fact that the solution of a given stage will define the level of optimality of solutions on its following stages. Moreover, trying to optimize the overall problem solution needs a review of all token decision and not a solution of particular sub-problem in a particular level.

3 The DAG traversal algorithm

The directed acyclic graph generated by modeling the master problem using the hierarchical modeling framework should be traversed in a systematic way to visit all nodes. Visiting a node in the graph consists of solving the sub-problem present there that may lead to recursively solve all sub-problems components of the current node. This will lead to the observation that applying a solving algorithm to the problem in a node is possible only for leaf nodes. Moreover, the solution generated at a particular node will be back propagated to source nodes depending on the type of linking edges. For instance, if it's decomposition edge then the solution of the current node (sub-problem) will be used to construct the solution of the source sub-problem. And, if it's a dependency edge then it (solution of the current node) will be used to update the instance (data) of the source node. Consequently, and in order to solve all nodes the nodes of the dependency edges should be visited before the nodes of the decomposition edges.

4 The solving approach

To reach an optimal solution for the HCSP, three solving approach will be used to solve the HCSP components which are as follows:

The grouping component will be solve using the K-means ++ algorithm.

The assignment component will be solved using the Hungarian algorithm.

The routing component will be solved using the Tabu Search meta-heuristic.

A set of experiments will be then conducted on real life data instances to validate our modeling and solving approach which will be compared to the work of the Rasmussen et al. [1].

5 Conclusions

When dealing with the solving of a combinatorial optimization problem, the general methodology is to analyze the problem. Then, we have to look for an adequate OR classic technique to be adapted and applied. This research deals with the solving of a combinatorial optimization problem which is the HCSP using a general Directed Acyclic Graph traversal. In this paper, we defined the graph traversal algorithms and we justify the interest grant to this algorithm by their theoretical and practical insights. Then, we detailed our proposed algorithm by presenting the code fragment of the algorithm for performing the traversal of a DAG. The algorithm was then analyzed by the precision of its properties such as the complexity, the optimality, the completeness, etc. As a future work, we aim to apply our modeling and solving approach to a complex optimization problem in the field oh health care which the Home Care Scheduling Problem by conducting a set of experiments on real life instances.

References

1. Rasmussen, M.S., Justesen, T., Dohn, A., Larsen, J.: The home care crew scheduling problem: Preference-based visit clustering and temporal dependencies. *European Journal of Operational Research*,2012; 219,598-610.