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A hierarchical decomposition framework for modeling combinatorial optimization problems

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Abstract

Complex Optimization Problems has existed in many fields of science, including economics, healthcare, logistics and finance where a complex problem has to be solved. Thus, modeling a complex problem is a fundamental step to relax its complexity and achieve to a final solution of the master problem. Hierarchical optimization is a main step in optimization problems handling process. It consists of decomposing an optimization problem into two or more sub-problems; each sub-problem has its own objectives and constraints. It will help to prove the correct understanding and represent the problem in a different form that facilitates its solving. In this work, we stipulate that a hierarchical decomposition of complex problems can yield to more effective solutions. The proposed framework will contain four possible strategies which will be detailed through this paper; objective based decomposition; constraints based decomposition, semantic decomposition and data partitioning strategy. Each strategy will be argued by a set of examples from the literature to validate our framework. However, some conditions shall be verified to model the problem using such conditions are problems' characteristics that will help to identify if a combinatorial optimization problem can be modeled within the proposed framework and they are detailed in the following subsections.

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Keywords: Hierarchical optimization; objective based decomposition; constraints based decomposition; semantic decomposition; data partitioning strategy; large scale data; data category.

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1. Introduction

Hierarchical optimization consists of decomposing an optimization problem into two or more sub-problems; each sub-problem has its own objectives and constraints. These sub-problems are usually interconnected in a hierarchical structure where a sub-problem in level i coordinates with a sub-problem of level i-1. The final solution of the main problem is produced by combining in some way the solutions of different sub-problems. Hierarchical optimization can be viewed as an application of the divide and conquer strategy for handling complex and hard optimization problems. The objective is to represent hard problems as a set of interconnected sub-problems. The links and relationships between sub-problems define the integration schema that will be used to combine partial solutions to obtain the final solution of the initial problem. The integration schema and the nature of links between sub-problems show simply the hierarchical structure proposed to represent the initial problem as a set of sub-problems. In the literature, we can find different and interchangeably used terminology like: Hierarchical optimization[1]; Multi-level optimization[2] and Dynamic optimization[3]. Recently, hierarchical optimization techniques have been applied to model and solve many optimization problems arising in many fields including: transportation[4]; business, economics and finance[5], Energy planning[6,7], industrial production planning[8], healthcare problems[9]. Modeling and solving above complex and large size problems as hierarchical optimization problem helps to reduce the time, reduce the sub problems search spaces, increase the performance and the exibility of solving algorithms and reduce the implementation cost. In this research paper, we will present the literature on hierarchical optimization. We will draw a clear framework that will help to classify hierarchical optimization problems. The proposed framework is an alternative for modeling complex optimization problems as a set of easier sub-problems. But, two main questions should be answered: when can a complex problem be decomposed into 'smaller' sub-problems and how to identify the required sub-problems? Many characters aid us to precise if a particular problem can be considered as a hierarchical problem or not. Let's remember that a decomposition modeling form is invented to facilitate the resolution of complex and difficult problems. Generally, complex problems need to be modeled hierarchically to facilitate their resolution. This study aims to answer the above central questions. We identified four characteristics of a particular complex problem to be modeled using the hierarchical optimization framework, which are:

- Multi-objective problems.
- Over-constrained problems.
- Very large instances.
- Problems with partial nested decisions.

We will present complex problems' decomposition strategies and argue them by a set of examples of hierarchical problems. Four decomposition strategies will be presented. First, the objective-based decomposition strategy where sub-problems are derived by separating different objectives and a sub-problem will be defined for each objective. The second strategy is the semantic decomposition which is based on decomposing the initial problem into a set of semantically linked sub-problems. The idea is to convert the main problem into a set of well-known or already solved canonical optimization problems like the Knapsack problem, the Traveling Salesman Problem, etc. The constraints relaxation strategy consists of relaxing complicated constraints to obtain easier sub-problems to solve. In this class, we can mention the well-known Branch and Bound algorithm which is based on the relaxation of the integrity constraints and defines at each level (iteration) a less complicated sub-problem. The fourth strategy is based on data partitioning where the instance to solve is organized as a collection of data subsets which will be resolved recursively or iteratively by applying the same process at different stages. This paper will be organized as follows: In the next section, we will present the outcomes and benefits of hierarchical optimization modeling. Section 3 will be devoted to detail the necessary conditions to model an optimization problem using the proposed hierarchical framework. The following section will detail the decomposition strategies and the associated literature. The strategies presented in section 4 will be supported by a set of examples in section 5. The paper will then be concluded and some future research perspectives will be addressed in the last section.

2. Motivations and benefits

In this paper, we propose a new modeling technique for complex optimization problems; it is based on the application of the Divide and Conquer strategy. The modeling process aims to identify a set of sub-problems interconnected in such a way to represent all the requirements of the main problem. The proposed optimization problems modeling alternative permits the following benefits:

2.1. Multidisciplinary

Large-scale problems require multidisciplinary decision making at multiple levels of a decision hierarchy. The multilevel optimization facilitates the modeling of problems in which different disciplines interact. It is well advised to model problems like hierarchical optimization problem where a number of engineering disciplines interact in order to obtain an integrated optimum model. Hierarchical optimization allows designers to incorporate all relevant disciplines simultaneously.

2.2. Reusability

After decomposing the principal problem, the resulting sub- problems can be resolved iteratively or recursively by applying the same process at different levels (on different data sets). The reusability of the toolbox of programs to resolve the sub-problems guarantee the consistency, extensibility, modularity, exibility, etc.

2.3. Reduction of search space

By decomposing the initial problem into a set of sub-problems we will intuitively transform the initial, generally very large search spaces into a reduced search spaces. As stated by Shobana et al.[10], the identification of intermediate sub-problems which decompose a problem can significantly reduce search and empirical evidence of the net benefit.

2.4. Parallel processing

The parallel processing is the ability to carry out multiple operations or tasks simultaneously. The multilevel optimization allows parallel processing in which sub-problems can be solved in the same time in a parallel computing environment to guarantee high-performance computing.

2.5. Time minimization

The multilevel optimization consists of solving a set of sub-problems and then combining the obtained partial solutions to find initial problem global solutions. The sub-problems are supposed to be easier to solve than the initial problem; thus the required time to solve each sub-problem separately and then integrate partial solutions will be significantly less than the time required for solving the initial problem as a unit.

3. Decomposability conditions

Problems' modeling is the main step in optimization problems handling process; it will help to prove the correct understanding and represent in a different form that facilitates its solving. In this work, we stipulate that a hierarchical decomposition of complex problems can yield to more effective solutions. However, some conditions shall be verified to model the problem using the Hierarchical framework. Such conditions are problems' characteristics that will help to identify if a combinatorial optimization problem can be modeled within the proposed framework and they are detailed in the following subsections.

3.1. Condition 1: Multi-Objective Problem

Multi-objective Optimization Problems has been existing in many fields of science, including healthcare, economics, finance and logistics where a complex problem has to be solved. Generally, multi-objective problems aim to realize multiple and often conflicting objectives to be optimized. Consequently, a solving approach for a multi-objective optimization problem should provide a set of solutions with the best compromise between all required objectives. Optimizing with multiple different and conflicting objectives is an additional level of complexity of optimization problems. Then, it is possible to decompose the master problem into two or more sub-problems based on objectives; each sub-problem has its own objectives and constraints. To illustrate this idea, let's cite the work of Begur et al.[11] in which authors studied the Home Health Care Scheduling Problem and considered it like a multi-objectives problems that aims to satisfy three objectives to know:

- The first objective is to assign patient visits to specific weekly time during a 16-or-so-week horizon,
- The second objective is to allocate the visits planned for a given patient to a specific day of the week,
- The third objective is to assign the patient visits scheduled for a given day to a particular nurse.

3.2. Condition 2: Over constrained problems

In many real-life applications (logistics, transportation, finance ...) most optimization problems are highly constrained where different types of constraints (resources, technical, etc) have to be satisfied in the final solutions. Over constrained problem are complex and difficult to solve like a single monolithic problem. To handle this problem, a constraint relaxation mechanism becomes necessary which consist to approximate of a difficult problem by nearby problems that are easier to solve by relaxing complicating constraints and recalling them after. Thus, the initial complex problem is modeled and solved in multiple levels, in each level a set of constraints will be satisfied until gratifying all required (hard) constraints and as much as possible satisfying preferential (soft) constraints. In this context, constraint hierarchies is a new concept proposed to describe high constrained problems by specifying constraints with hierarchical strengths or preferences, i.e. required and preferential constraints, most important and less important (preferences) constraints, big penalty violation and small penalty violation, moreover, constraint hierarchies allow "relaxing" of constraints with the same strength by applying, e.g., weighted-sum, least-squares or similar comparators [9].

3.3. Condition 3: Conditions on data

Two data sets characteristics can help to define the main problem to solve can be modeled using the proposed hierarchical framework. First, if the size of the data set of the instance is very large then it will be possible to divide it into two or more subsets based on particular criterion (geographic for example). Second, in some problems the data is classified into different types (level of proficiency of nurses, level of patients' illness, etc). For such problems the data sets can be partitioned following the defined data types. In the following we detail each condition and give some illustrative examples.

3.3.1. Large scale data sets

Large scale data sets are collections of so large and complex data inputs that it becomes difficult to process traditionally as one batch. With very large data sets, experiments may face ambiguous situations and may not end. According to Andrew and John[12], data decomposition is the other primary form of breaking up monolithic processing into chunks that can be farmed out to multiple cores for parallel processing. The size of the problem space is one of the most obvious candidate measures of complexity which involve modeling a particular problem hierarchically. Problem difficulty was thought to vary with the size of the problem space. Hertz and Lahrichi[13] presented an illustration of data-partitioning decomposition strategy to model and solve the huge size of the problem space of the home care scheduling problem. Considering the very large size of the Canadian territory and to balance the work load of nurses while avoiding long travels to visit the clients, authors partitioned the Canadian territory into 6 districts.

3.3.2. Categorical data

Another attribute of data sets of complex problem that can be modeled within the hierarchical framework is the existence of types and categories. Categorical data divide implicitly the input data into classes like types of customers in banking (Very Important Persons, Important, Ordinary) or type of employees following their skills (Expert, Skilled, Basic), etc. Such characteristics help to organize the main data set into smaller subsets following the proposed categories which define a set of sub-problems. The main problem will be consequently solved via solving each component (sub-problem separately) and then merging the obtained partial solutions to form the main solution. Hertz and Lahrichi[13] propose to decompose the instance of the Home Health Care Scheduling Problem, following categories of patients and nurses. Similarly, Mullinax and Lawley[14] decompose the data set into three sub groups based on the patient level of illness (state), into patients that require minimal care; patients that require close attention and critically ill patients.

3.4. Condition 4: Problems with partial nested decisions

Generally, complex problems are multi-decision problems where some intermediate decisions must be taken to reach a final solution of the main problem. Multi-decisions problem solving process embeds the solving of sub-problems at different times by different decision makers at different levels. The final solution will be built then by combining in some way the partial solutions of intermediate sub-problems. Consequently, the structure of the initial problem can be modeled as a particular combination of sub-problems. Such sub-problems are intuitively easier to handle and to solve than the main problem for different reasons: reduced search space and data sets, uncomplicated combinatorial structure, adapted solving approaches may be already known and solving tools (software) are available. For the above advantages, it is possible to represent multi-decisions problems by its components (sub-problems) organized in such a way to fulfill the requirements of the initial problems. For instance, consider the Home Health Care Scheduling Problem where the question is about scheduling to serve patients at their home subject to different types of constraints and optimizing some objectives. Finding such a solution for the Home Health Care Scheduling Problem, passes through determining which nurse will serve which patient, then how their medical teams will be formed to move together and finally which routes will be followed to reach patients' homes in the transportation network [9].

4. Problem decomposition strategies

As mentioned in the introduction of this paper, it is necessary to know how to decompose a complex optimization problem into a set of interconnected sub-problems and what are the nature of links and relationships between them. In this section we detail four possible strategies for splitting the main problem into a set of different components. The four strategies are presented in the following figure.

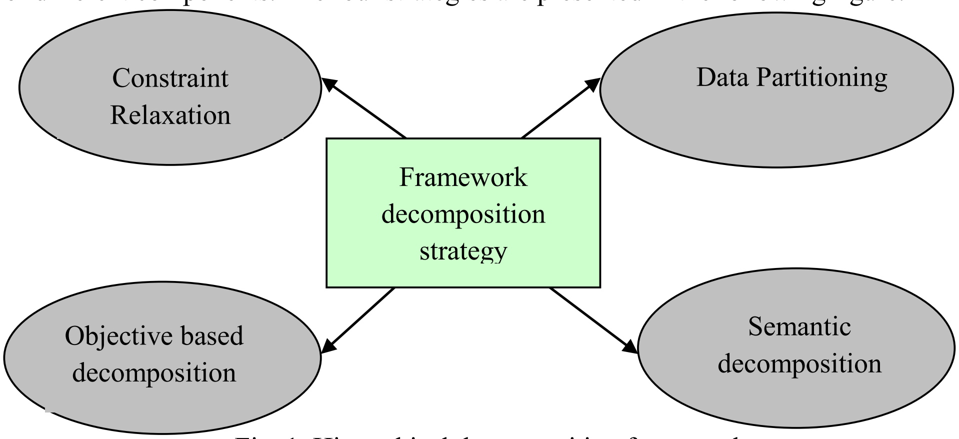

Fig. 1. Hierarchical decomposition framework

4.1. Objective-based Decomposition

The decomposition by objectives consists of dividing the basic problem into a set of sub-problems based on targets (objectives) of the initial problem. For each problem at a given level, an optimization sub-problem is formulated to satisfy a set of constraints and achieve a certain objective function. The decomposition by objectives is a very useful and powerful tool to model large scale problems in a hierarchical structure. To develop the proposed strategy, we propose a four steps algorithm which consists to specify first the problem targets, and then propagate such objectives to build their corresponding sub-problems. Then, the components achieve their respective targets and the resulting model meets overall problem targets. Supposing that we have two predefined functions, with a particular component (sub-problem) (P) in the input:

- Decomposable (P) aims to determine if P is decomposable or not.
- Solve (P) aims to solve P.

Given a main complex P*i* with an Objective O to realize and a set of constraints C to satisfy, and let S*i* the solution of the sub-problem P*i*, the pseudo code is summarized in (Algorithm 1).

Algorithm1:Objective_Based_Decomposition_Strategy (P)
Input: A main complex problem P {O, C} to solve
Output: A solved problem
BEGIN
Step1: Specify problem (n) targets ($\sum_{i=1}^{n} O_i$ *)*
Step2: Assign each target to a sub-problem (P= $\sum_{i=1}^{n} P_i$ *)*
Step3: For (i=1; i<=n; i++)
{ If (Pi is decomposable(Pi))
{Objective_Based_Decomposition_Strategy (Pi);}
Else {Solve (Pi);}
Step4: Combining solutions of Pi (S= $\sum_{i=1}^{n} S_i$ *);*
End

In this context, Mutingi and Mbohwa[15] modeled and developed a robust support tool using the objective based decomposition strategy in which they aims to satisfy three conflicting management goals. The initial problem was hierarchically decomposed into three sub-problems, each one with its objective and constraints. The first sub-problem aims to minimize the schedule cost associated with the trips which is influenced by the nature of the routes assigned to healthcare workers to fulfill the demand requirements. The second component aims to maximize worker satisfaction which entails meeting the worker preferences to the highest degree possible and especially by ensuring the fairness in workload. The last sub-problem concerns the maximization of client satisfaction which can be ex- pressed as a function of the violations of time windows preferred by the clients. The multi-objective formulation is achieved by optimizing the three objective functions jointly and the three components were solved in a parallel processing form.

4.2. Semantic decomposition

Many practical optimization problems involve some intermediate decisions to be taken during the solving process. We refer to such intermediate decision as nested decision that are solving also intermediate problems. Intermediate sub-problems are easier to solve than the initial problem and the quality of their solution affect the quality of the solution of the main problem. Generally, sub-problems are canonical and known problems and sometimes their solving approach is well known and validated. For instance, let's consider the distribution of goods and products in a transportation network; this problem is based on the available fleet of trucks. Suppose that the company carrying this activity is willing to choose the set of trucks to buy and then to use depending on their capacity, speed and price. It's clear here that distribution task and its solution depends on the bought trucks i.e. the solution of the trucks buying problem affect considerably the solution of the distribution problem. The Semantic decomposition strategy aims to isolate embedded sub-problems and their models from the initial problem; solve them and then rebuild the big solution from the obtained partial (intermediate) solutions. Semantic decomposition consists of dividing an initial large scale and hard optimization problem into a set of semantically independent sub-problems. Many well-known combinatorial optimization problems have been modeled as hierarchical problems using the semantic decomposition. The Machine scheduling problem consists of assigning a set of jobs to a machine in such a way that the capacity constraint is not violated. It is easy to remark that such a constraint is of the same form as that of a knapsack problem. Another important example in which knapsack problems arise is the capital budgeting problem. This problem involves finding a subset of the capital projects under consideration that will yield the greatest return on investment, while satisfying specified financial, regulatory and project relationship requirements. Airline fleet assignment problems are also complex optimization problems that consists of determining the staff required to ensure predefined flights. The sub-problems are the team building problem (which members will travel together?) and the assignment of the already built teams to flights. The home health care scheduling problem has been also decomposed semantically into three components: assignment problem (Which caregiver will serve which patient?), Grouping problem (build the teams that will move together) and a routing problem (design the routes to be followed by the medical teams). In the following, we present an illustrative schema for the modeling of the Home Health Care Scheduling Problem using the semantic decomposition strategy. In the following we illustrate the semantic decomposition strategy by an illustrative example:

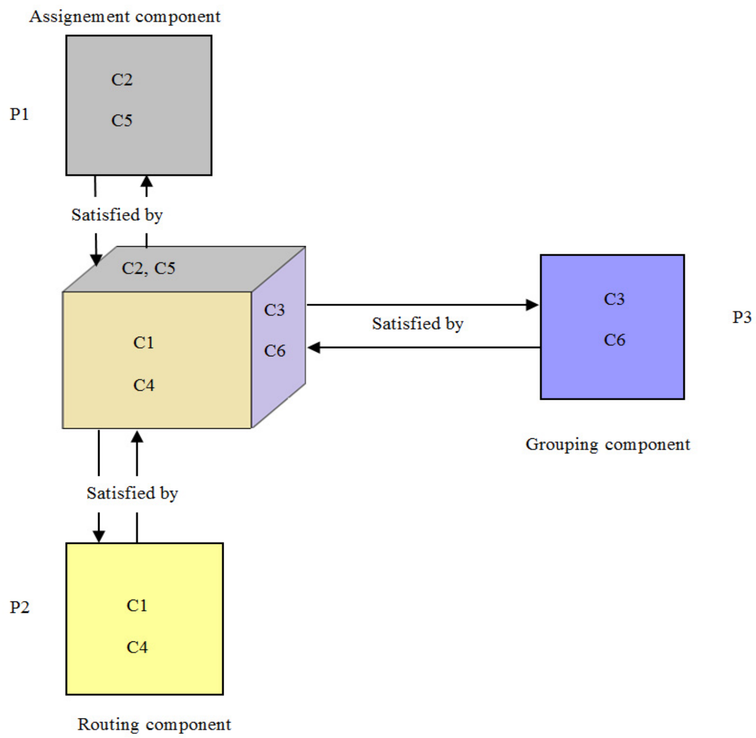

Fig. 2. Illustrative example of the semantic decomposition strategy

4.3. Constraint Relaxation

Many real-life problems are over-constrained (logistics, transportation, finance...). To handle this kind of highly constrained problems, a constraint relaxation mechanism becomes necessary. A solution of the relaxed problem provides information about the original problem. This approach consists of relaxing a set of complicating constraints in order to obtain a more tractable model. By removing the complicating constraints from the constraint set, the resulting sub- problem is frequently considerably easier to solve. This technique requires that one understands the structure of the problem being solved in order to then relax the hard constraints. Many features of constraint relaxation can be detected like relaxing the integrity of decision variables. Another example is the Lagrangian relaxation of a complicated problem in combinatorial optimization which penalizes violations of some constraints, allowing an easier relaxed problem to be solved. In the following we illustrate the constraints relaxation decomposition strategy by an illustrative example:

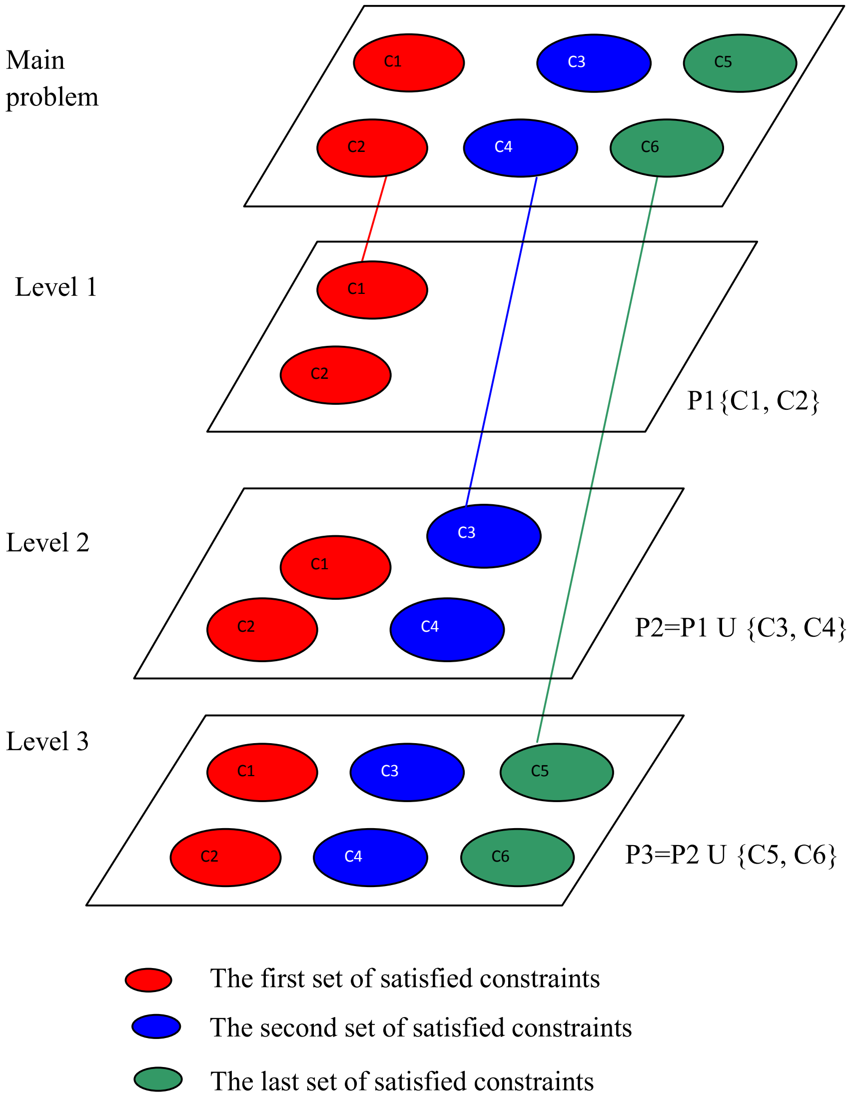

Fig. 3. Illustrative example of the constraints relaxation strategy

4.4. Data Partitioning

This strategy consists in partitioning the global problem instance to solve into a collection of data sets to reduce the complexity of the original problem that is usually large scale and difficult to solve in one track. It may be easier to organize the problem as a collection of data sets with well-defined relations rather than attempt to pose a single monolithic problem. The resulting data sets will be resolved iteratively or recursively by applying the same process at different data subsets. The work of Hertz and Lahrichi[13] present a good illustration of the data partitioning strategy in modeling a problem with a very large size (the Canadian territory), which was partitioned into 6 districts {A, B, C, D, E, F}. See figure (4).

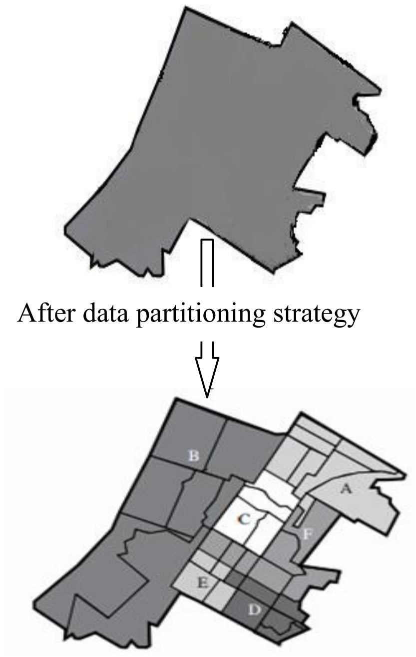

Fig. 4. The six districts of the Canadian territory

5. Conclusion and perspectives

The hierarchical decomposition frameworks to model complex optimization problems are based on their decomposition into a set of interconnected sub-problems easier to handle. It's an application of the Divide and Conquer strategy to facilitate the handling of difficult problems. We detailed the necessary conditions to model an optimization problem using the proposed framework. The proposed four problems decomposition strategies based on objectives, constraints relaxation, data partitioning and semantically. The set of derived sub-problem should be linked and their partial solution should participate to build the final solution of the main initial problem. The proposed modeling approach can be applied efficiently to solve many kinds of optimization problems particularly those where the solutions are nested and not trivial to found such the Home Health Care Scheduling Problem. In the forthcoming project, we will attempt to model the home health care problem using the proposed framework and detailing the possible relationships that may link two sub-problems within the framework.

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