Multi-source possibilistic influence diagrams considering confidence degree and similarity measure

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***Abstract*—This paper proposes a new variant of quantitative possibilistic influence diagrams which are extension of standard influence diagrams in the possibilistic framework. We will in particular treat the case where several expert opinions relative to value nodes are available. Among these experts, there is an initial expert who assigns confidence degrees to other experts and who fixes a similarity threshold that provided possibility distributions should respect. An evaluation algorithm for these multi-source possibilistic influence diagrams will also be proposed.**

I. INTODUCTION

These last decades knew intense political and economic changes which increases the uncertainty lied to real problems and makes decision making a very delicate task. In this context, graphical decision models allow a compact and a simple representation of decision problems. Within most popular decision models, we can mention decision trees [14], influence diagrams [11] and valuation based systems [18].

In this paper, we are interested, in particular, in influence diagrams (IDs) which provide efficient decision tools under uncertainty. An ID is composed by a graphical component which is a Direct Acyclic Graph (DAG) and a numerical one quantifying this DAG. The numerical component is generally provided by experts who will express their uncertainty relative to variables (represented by chance nodes) via probability distributions and their preferences (represented by value nodes) through utilities. Nevertheless, in most real problems it is not obvious to provide exact probability distributions and it is easier to express uncertainty in a qualitative manner. Moreover, decision makers (DM) may encounter several difficulties when expressing their utilities.

We have already attacked these problems by proposing what we have called *possibilistic influence diagrams* [7] [8] [9] which are possibilistic counterpart of standard influence diagrams where the numerical component is modeled in the possibilistic framework [4] which offers an alternative choice to the probabilistic one with a particular ability to handle uncertain information in a qualitative manner.

In this paper, we propose another variant of possibilistic influence diagrams considering the case of several expert opinions in order to improve the quality of utilities. More precisely, we develop the case where an initial expert will propose a set of possible numerical utilities and a possibility distribution relative to each consequence and each utility. Then, a group of assistant experts will help him by providing, in their turn, their opinion regarding the same elements. Then, the initial expert will express his opinion concerning the reliability of assistant experts by assigning confidence degrees to each of them. In addition, he should provide a similarity threshold which will be taken into account in the evaluation process. In fact, only possibility distributions with similarity measure higher than the fixed threshold will be considered. Then, the conjunction between possibility distributions relative to each utility and each consequence will be computed in order to obtain one possibility distribution.

Once this step achieved, we will have a possibilistic influence diagram quantified in the same way than those presented in [8] which means that the already developed evaluation method can be used in order to generate best strategies.

This paper is organized as follows: Section 2 provides the necessary background on possibility theory. Section 3 presents possibilistic influence diagrams. Section 4 proposes the new variant of possibilistic influence diagrams involving several experts. Finally, Section 5 details the evaluation algorithm of multi-source possibilistic influence diagrams.

II. BACKGROUND ON POSSIBILITY THEORY

Possibility theory was proposed as an alternative theory of uncertainty in order to remedy the incapacity of probability theory for modeling total ignorance and qualitative uncertainty. This theory was initially proposed by Zadeh [20] and was developed by Dubois and Prade [4]. This section briefly recalls basic elements of possibility theory, for more details see [4].

The basic buildings block in the possibility theory is the notion of possibility distribution denoted by π, it is a mapping from the universe of discourse denoted by $\Omega = \{\omega_1 ... \omega_n\}$ to the unit interval $[0, 1]$. This scale has two interpretations, a quantitative one when the handled values have a real sense and a qualitative one when the handled values reflect only an ordering between the different states of the world. In the first case, the *product operator* can be applied while in the second one, the *min operator* is used.

In the possibilistic framework, extreme forms of partial knowledge can be represented by *complete knowledge* i.e. $\exists \omega_i \in \Omega$, s.t $\pi(\omega_i) = 1$ and $\omega_j \neq \omega_i, \pi(\omega_j) = 0$ and the *total ignorance* i.e. $\forall \omega_i \in \Omega, \pi(\omega_i) = 1$.

A possibility degree is a value from the interval $[0, 1]$ associated to each element ω of Ω. The possibility measure of any subset $\psi \subseteq \Omega$ is defined as follows:

$$\Pi(\psi) = max_{\omega \in \psi} \pi(\omega) \quad (1)$$

Π has a dual measure which is the necessity measure

A possibility distribution is said to be *normalized*, if $max_{\omega \in \psi}\pi(\omega) = 1$ (Namely, ω is a totally possible state) and it is said to be *sub-normalized* if $max_{\omega \in \psi}\pi(\omega) \neq 1$.

The value $1 - max_{\omega \in \psi}\pi(\omega)$ is called *the degree of inconsistency* of a possibility distribution and it is commonly used to measure the level of conflict in information issued from multiple sources after merging them.

Namely, if $1 - max_{\omega \in \psi}(\pi_1(\omega) \wedge \pi_2(\omega)) \neq 0$ then $\pi_1 \wedge \pi_2$ is a sub-normalized possibility distribution and there is a conflict between π_1 and π_2.

Given two possibility distributions from two distinct sources of information, then it is possible to compute the similarity between these two sources through a similarity measure between the two possibility distributions. In the literature, several similarity measures have been proposed such as *Manhattan Distance* and *Euclidean Distance* which satisfy basic properties of any possibilistic similarity measure [12].

In the possibilistic approach, there are several combination modes to ensure the information fusion and the choice of the appropriate method is related to the reliability of information's sources.

The most known combination operators are the symmetric ones, namely the *conjunctive* and the *disjunctive* operators:

1) **The conjunctive fusion:** If all sources are reliable, then we can combine them using the intersection, the conjunctive operator $\bigotimes$ is defined as follows:

$$\forall\ \omega \in \Omega, \quad \pi_{\wedge}(\omega) = \bigotimes_{i=1..n} \pi_i(\omega) \tag{2}$$

   where π_i be the possibility distribution supplied by source i.
   $\bigotimes$ is a t-norms such that minimum or product or linear product according to the uncertainty scale's interpretation. Indeed, the min operator is supported by both quantitative and qualitative possibility distributions.
   However, the use of the product operator assumes that possibility degrees are numerical.

2) **The disjunctive fusion:** This mode of combination is applied when it is known for sure that at least one of the sources is reliable but it is not known which one. The disjunctive operator $\bigoplus$ is defined as follows:

$$\forall\ \omega \in \Omega, \quad \pi_{\vee}(\omega) = \bigoplus_{i=1..n} \pi_i(\omega) \tag{3}$$

   $\bigoplus$ is a t-conorms such that maximum or probabilistic sum or Lukasievicz according to the uncertainty scale's interpretation. Indeed, all of these t-conorms can be applied in the quantitative setting. However, only the maximum operator can be applied in the qualitative setting.

The conditioning represents a special case of information fusion. Indeed, it consists in revising our initial knowledge, represented by a possibility distribution π, which will be changed into another possibility distribution $\pi' = \pi(.|\psi)$ with $\psi \neq \emptyset$ and $\Pi(\psi) > 0$.

The two interpretations of the possibilistic scale induce two definitions of the conditioning:

- Min-based conditioning relative to the ordinal setting:

$$\pi(\omega|_m\psi) = \begin{cases} 1 & if\ \pi(\omega) = \Pi(\psi)\ \ and\ \ \omega \in \psi \\ \pi(\omega) & if\ \pi(\omega) < \Pi(\psi)\ \ and\ \ \omega \in \psi \\ 0 & otherwise \end{cases} \tag{4}$$

- Product-based conditioning relative to the numerical setting:

$$\pi(\omega|_p\psi) = \begin{cases} \frac{\pi(\omega)}{\Pi(\omega)} & if \omega \in \psi \\ 0 & otherwise \end{cases} \tag{5}$$

III. POSSIBILISTIC INFLUENCE DIAGRAMS

Few works exist on possibilistic networks and existing ones concern reasoning under uncertainty without considering the decision aspect [1], [2].

Recently, Sabbadin et al. [5] have proposed possibilistic influence diagrams using *optimistic and pessimistic utilities* [4] for the quantification of value nodes. Nevertheless, Giang et al. [6] noted that this utility framework is based on axioms relative to uncertainty attitude contrary to the VNM axiomatic system [13] based on risk attitude, which does not make a sense in the possibilistic framework since it represents uncertainty rather than risk. Moreover, to use pessimistic and optimistic utilities, the decision maker should classify himself as either pessimistic or optimistic which is not always obvious. To overcome these limitations, Giang et al. [6] propose a more generalized framework based on the axiomatic system of *possibilistic binary utility*.

The theory of possibility offers a rich and effective framework for the representation and the treatment of the uncertainty, what motivated us to develop possiblistic influence diagrams which benefit also of the simplicity of standard influence diagrams.

Formally, a possibilistic IDs has two components:

1) **A graphical component** defined by a directed acyclic graph (DAG), denoted by $G(N, A)$, where N is the set of chance, decision and value nodes and A is the set of arcs in the directed graph.
2) **A numerical component** evaluating different dependencies between chance nodes and utilities for value nodes.
   - For each chance node C_i, we should provide conditional possibility degree $\Pi(c_{ij} \mid pa(C_i))$ of each instance c_{ij} of C_i in the context of each instance of its parents. In order to satisfy the normalization constraint, these conditional distributions should satisfy, $\forall pa(C_i)$:

$$max_{c_{ij}}\Pi(c_{ij} \mid pa(C_i)) = 1, \tag{6}$$

     Note that for root chance nodes (i.e. $(Pa(C_i) = \emptyset)$, equation (6) corresponds to $max_{c_{ij}}\Pi(c_{ij}) = 1$.
   - For each value node V_i, there are several ways to represent decision maker's preferences on the set of consequences, namely using cardinal utility, ordinal utility, possibilistic utility or as well as a compound utility.
     Note that likewise standard IDs, decision nodes in possibilistic IDs are not quantified.

Different combinations between the quantification of chance and utility nodes offer several kinds of possibilistic IDs which can be regrouped into three principal classes:

- *Product-based possibilistic IDs* where both dependencies between chance nodes and value nodes are quantified in a genuine numerical setting.
- *Min-based possibilistic IDs or qualitative possibilistic ID* where both dependencies between chance nodes and value nodes are quantified in a qualitative setting used for encoding an ordering between different states of the world.
- *Mixed possibilistic IDs* where dependencies between chance nodes and value nodes are not quantified in the same setting.

Product-based and min-based possibilistic IDs represent *homogeneous* possibilistic IDs and mixed possibilistic IDs are the *heterogeneous* ones.

In a previous study, we have developed *min-based possibilistic IDs* where for the quantification of value nodes, we have handled two cases: the first one concerns the use of the ordinal utility [7] and the second is about the application of the qualitative binary possibilistic utility [9].

In addition, we have developed *product-based possibilistic IDs* where for the quantification of value nodes, we have handled two cases: the first one concerns the use of cardinal utility [9] and the second one treat the case where decision makers provide a set of numerical utilities and a possibility distribution relative to each consequence and each utility [8].

To evaluate these possibilistic IDs, we have proposed an indirect evaluation method, in which we have used the information fusion in the possibilistic framework. In fact, the product operator was used for the conjunction and the max operator was used for the disjunction. This paper proposes an extension of these IDs to the case of several experts as detailed in next section.

IV. Multi-source possibilistic influence diagrams

The new variant of possibilistic IDs proposed in this paper deals with the case of several experts that will collaborate for the quantification of value nodes. Indeed, in these IDs dependencies between chance nodes will be expressed by quantitative possibility distributions and value nodes are quantified by several experts.

In fact, an initial expert E_0 and n assistant experts (E_i where $i \in \{1..n\}$) will model their uncertainty relative to each value node.

More precisely, the initial expert will provide a set of m numerical utilities, denoted by UT and also a possibility distribution relative to each utility and each consequence (without affecting the exact value of utility to the appropriate consequence). Then, assistant experts will help initial expert to improve the accuracy of his knowledge concerning value nodes and they will provide, in their terms, possibility distributions relative to each utility and each consequence.

The possibility distribution provided by the initial expert will be denoted by Π_0, and this expressed by the assistant expert E_i by Π_i. These possibility distributions should satisfy, $\forall x \in X$:

$$max_{UT_j \in UT} \Pi_i(U(x) = UT_j) = 1 \quad \forall i \in \{0..n\} \qquad (7)$$

To avoid unreliable opinions and possible contradictory knowledge, the initial expert will assign a confidence degree to each assistant expert. These degrees reflect the confidence of the initial expert in each assistant expert. The confidence degree relative to the assistant expert E_i is denoted by α_i such that $\alpha_i \in [0, 1]$. We can distinguish the following different cases concerning α_i:

- if $\alpha_i = 0$ then E_0 has no confidence in E_i.
- if $\alpha_i = 1$ then E_0 has a total confidence in E_i.
- if $\alpha_i = 0.5$ then E_0 is neutral concerning E_i.
- if $\alpha_i \in]0, 0.5[$ then E_0 isn't confident in E_i.
- if $\alpha_i \in]0.5, 1[$ then E_0 is confident in E_i.

In addition, the initial expert will fix a similarity threshold (denoted by TH), beyond it possibility distributions are considered in contradiction with his knowledge. $TH \in [0, 1]$.

According to the value of TH, we can classify the behavior of the initial expert as follows:

- if $TH = 0$ then E_0 has a pessimistic behavior characterized by uncertainty aversion.
- if $TH = 1$ then E_0 has an optimistic behavior characterized by uncertainty attraction.
- if $TH = 0.5$ then E_0 has a neutral behavior.

Example 1: Let us consider the simple decision problem represented by the possibilistic ID of figure 1 containing 3 chance nodes (A, B, C), 1 decision node (D) and 1 value node (V). Possibility distributions for the chance nodes are presented in table I.

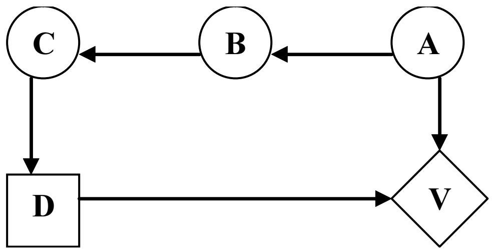

Fig. 1. An example of influence diagram

TABLE I
A PRIORI AND CONDITIONAL POSSIBILITY DISTRIBUTIONS FOR CHANCE NODES

A	$\Pi(A)$	A	B	$\Pi(B\|A)$	B	C	$\Pi(C\|B)$
T	1	T	T	0.9	T	T	1
F	0.6	F	T	0.2	F	T	0.3
		T	F	1	T	F	0.2
		F	F	1	F	F	1

For the utilities, the initial expert affirms that the possible values of utilities are {4, 7, 8}.

For the sake of simplicity we will denote $(U(A, D) = 4)$ by U_1, $(U(A, D) = 7)$ by U_2 and $(U(A, D) = 8)$ by U_3.

The initial possibility distribution Π_0 relative to each consequence and utility provided by the initial expert is represented in table II:

To improve the precision of his uncertainty concerning the utilities, the initial expert will take into account the opinion of two other assistant experts (E_1 and E_2) who will provide possibility distributions Π_1 and Π_2 represented respectively in tables III and IV.

TABLE II
POSSIBILITY DISTRIBUTION $\Pi_0(U(A,D) = UT_i)$

A	D	$\Pi_0(U_1)$	$\Pi_0(U_2)$	$\Pi_0(U_3)$
T	d_1	0.2	1	0.3
F	d_1	1	0.1	0.2
T	d_2	0.6	0.1	1
F	d_2	1	0.1	0.3

TABLE III
POSSIBILITY DISTRIBUTION $\Pi_1(U(A,D) = UT_i)$

A	D	$\Pi_1(U_1)$	$\Pi_1(U_2)$	$\Pi_1(U_3)$
T	d_1	0.1	0.6	1
F	d_1	1	0.4	0.8
T	d_2	0.5	0.5	1
F	d_2	1	0.1	0.3

Suppose that the confidence degrees relative to assistant experts E_1 and E_2 are $\alpha_1 = 0.7$ and $\alpha_2 = 0.3$ which means that E_0 has a confidence in E_1 and not in E_2) and that the similarity threshold fixed by the initial expert is $TH = 0.3$ i.e. he has a pessimistic behavior.

V. EVALUATION OF MULTI-SOURCE POSSIBILISTIC INFLUENCE DIAGRAMS

Given a multi-source possibilistic ID, we should evaluate it in order to generate optimal decisions. As we have mentioned in the introduction, there are two approaches to evaluate standard IDs, namely, direct and indirect ones.

The evaluation of possibilistic IDs, proposed in [5], is based on an indirect evaluation method which transforms them into decision trees. Such evaluation method was not successful in the probabilistic framework since, contrary to those based on Bayesian networks, it does not use independencies encoded by IDs to save some computations since decision trees are not able to represent independencies [21]. This argument remains available in the possibilistic framework, as it only concerns the graphical component which is the same in the two frameworks.

In addition, direct evaluation methods [17] require heavy computations since they are based on arc reversal and node deletion, contrary to indirect ones which are based on the transformation of IDs into Bayesian networks. This explains the great development of indirect methods in the probabilistic case [3], [15], [16], [21].

The success of indirect evaluation methods for standard IDs, motivates us to develop an indirect evaluation method for possibilistic IDs. Our choice is reinforced by the fact that a possibilistic counterpart of Bayesian networks has been developed as well as their propagation algorithms [1].

More precisely, we will develop a possibilistic counterpart of Cooper's method [3] for the particular case of influence diagram with a unique value node, since it represents the basis of existing indirect methods.

Thus, the principle of our evaluation algorithm is to transform decision and value nodes into chance nodes in order to obtain a possibilistic network, and then to use this secondary structure to compute maximal expected utilities via a propagation process. These two major phases are detailed in what follows.

TABLE IV
POSSIBILITY DISTRIBUTION $\Pi_2(U(A,D) = UT_i)$

A	D	$\Pi_2(U_1)$	$\Pi_2(U_2)$	$\Pi_2(U_3)$
T	d_1	0.2	0.9	1
F	d_1	0.7	1	0.2
T	d_2	0.6	0.3	1
F	d_2	1	0.1	0.3

A. Transformation phase

This phase consists in transforming decision and value nodes into chance nodes.

1) Decision nodes transformation : Each decision node D_i in the possibilistic ID is transformed into a chance node which should be quantified. In the probabilistic case, this quantification is ensured by an equi-probable distribution. Nevertheless, this is not really appropriate, since equi-probability represents randomness rather than total ignorance. This problem can be overcome in the possibilistic framework where our ignorance about the new chance node can be suitably represented via a uniform possibility distribution. More formally:

$$\Pi(d_{ij} \mid_p pa(D_i)) = 1, \quad \forall d_{ij}, pa(D_i) \tag{8}$$

Example 2: The ID presented in figure 1 has one decision node D. The possibility distribution of the new chance node D obtained by equation (8). is presented in table V:

TABLE V
POSSIBILITY DISTRIBUTION $\Pi(D \mid C)$

C	D	$\Pi(D \mid C)$
T	d_1	1
F	d_2	1
T	d_2	1
F	d_1	1

2) Value node transformation : This phase starts by a processing step of possibility distributions given by several experts. The goal of this step is to consider only reliable opinions of assistant experts in order to avoid contradiction in knowledge. This processing step is detailed as follows:

1) For each E_i ($i \in 1..n$), compute the similarity between the initial possibility distribution Π_0 and his possibility distribution Π_i. This similarity measure will be denoted by $S_i(\Pi_0, \Pi_i)$ and can be computed by any quantitative similarity measure between two possibility distributions (see section 2).
   Let s_{ij} be the similarity measure between Π_0 and Π_i for the case of UT_j ($j \in 1..m$). Then, the similarity measure $S_i(\Pi_0, \Pi_i)$ will be the average of the all s_{ij} i.e.
   $$S_i(\Pi_0, \Pi_i) = \frac{\sum_{j=1}^{m} s_{ij}}{m}.$$
2) For each S_i ($i \in 1..n$), compute the weighted similarity measure i.e.
   $$S'_i = \alpha_i * S_i.$$
   This step aims to balance the similarity measure taking into account the confidence degree of the assistant expert.
3) Eliminate opinions whose relative weighted similarity measure are lower than the similarity threshold. Namely,

possibility distributions Π_i whose $S'(\Pi_0, \Pi_i) < TH$ will be eliminated.

4) Combine reserved possibility distribution Π_i with Π_0 using the min operator since it concerns a conjunctive fusion in quantitative setting. The resulted possibility distribution will be denoted by Π_r.

After this processing step, we will have a possibilistic influence diagram quantified in the same way that those yet developed in [8], so that the proposed value node transformation method can be directly applied.

Example 3: Let us continue with the same example. To measure the similarity between proposed possibility distributions, we will use the Manhattan Distance MD [12] as follows:

$$MD(\Pi_1, \Pi 2) = 1 - \frac{\sum_{i=1}^{n}(|\pi_1(\omega_i) - \pi_2(\omega_i)|)}{n}$$

Let S_1 (resp. S_2) be the similarity measure between Π_0 and Π_1 (resp. Π_0 and Π_2). Then for the computation of S_1, we have:

- s_{11} is the similarity measure between Π_0 and Π_1 in the case of U_1, namely where $U(A, D) = 4 \Rightarrow s_{11} = 0.95$.
- s_{12} is the similarity measure between Π_0 and Π_1 in the case of U_2, namely where $U(A, D) = 7 \Rightarrow s_{12} = 0.275$.
- s_{13} is the similarity measure between Π_0 and Π_1 in the case of U_3, namely where $U(A, D) = 8 \Rightarrow s_{13} = 0.325$.

Thus $S_1 = \frac{s_{11}+s_{12}+s_{13}}{3} = 0.516$. In the same manner we have $S_2 = 0.75$. Then, the weighted similarity measures are computed as follows:
$S'_1 = \alpha_1 * S_1 = 0.7 * 0.516 = 0.361$
$S'_2 = \alpha_2 * S_2 = 0.3 * 0.75 = 0.225$
Since $TH = 0.3$, then only the opinion of E_1 will be taken into account for the quantification phase

Once, this processing step is achieved, the conjunction of Π_0 with the reserved possibility distribution Π_1 will be computed using the min operator as shown in table VI.

TABLE VI
POSSIBILITY DISTRIBUTION $\Pi_r(U(A, D) = UT_i)$

A	D	$\Pi_r(U_1)$	$\Pi_r(U_2)$	$\Pi_r(U_3)$
T	d_1	0.1	0.6	0.3
F	d_1	1	0.1	0.2
T	d_2	0.5	0.1	1
F	d_2	1	0.1	0.3

To compute optimal strategy, the proposed algorithm [8] dealing with possibilistic influence diagrams using information fusion will be used. The obtained possibilistic network after the transformation of the decision node D and the value node V is presented in figure 2.

The first step for the transformation of the value node V in [8] is the transformation of numerical utilities into a possibility distribution as presented in table VII.

Each consequence has two information: $\Pi(V \mid A, D)$ and $\Pi_r(U_i)\ \forall i \in \{1, 2, 3\}$ which will be merged using the product operator. The result of this conjunctive fusion is Π_{V_i} presented in table VIII.

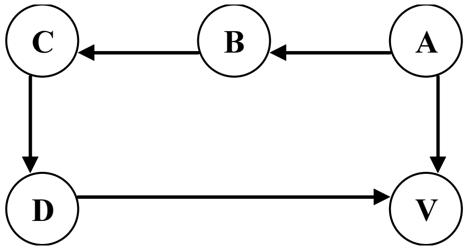

Fig. 2. Resulted possibilistic network

TABLE VII
TRANSFORMATION OF UTILITIES INTO A POSSIBILITY DISTRIBUTION

$UT(A, D)$	V	$\Pi(V \mid A, D)$
4	T	0
7	T	0.375
8	T	1
4	F	0
7	F	0.625
8	F	1

Since, the set of numerical utilities contains three possible values of utility then we will have three choices for each consequence (as presented in table VIII). For each consequence, the disjunctive fusion will be applied via the max operator. The possibility distribution issued by the disjunctive fusion is presented in table IX.

B. Propagation phase

The following step in the evaluation process of the possibilistic influence diagram is the propagation phase which consists in the use of the appropriate propagation algorithm in the possibilistic network issued from the transformation phase. The selection of the appropriate propagation algorithm depends of the DAG structure.

The propagation phase aims to compute the Maximal Expected Utility (MEU) relative to each decision node. This computation starts by the last decision node D_m to the first one D_1. Optimal decisions (i.e those relative to $D_1, .., D_{i-1}$) already computed should be integrated in the computation of the optimal decision relative to D_i. More formally, for each decision D_i, we have:

$$\Pi(D_i, E) = \Pi(v = T \mid Pa(V))\Pi(Pa'(V) \mid d_{ij}, E) \quad (9)$$

where E is the set of evidence and $Pa'(V)$ is the set of chance nodes in $Pa(V)$.

For the proposed possibilistic influence diagrams, we will always obtain a quantitative possibilistic networks, so product-based propagation algorithms in quantitative possibilistic networks should be used in order to compute $\Pi(Pa'(V) \mid d_{ij}, E)$.

Indeed, two product-based propagation algorithms have been defined according to the nature of the DAG in the possibilistic network [1]. More precisely, if the DAG is singly connected then the possibilistic adaptation of the centralized version of Pearl's algorithm should be used. Then, if the DAG is multiply connected then the possibilistic adaptation of junction trees propagation should be used.

Once, $\Pi(D_i, E)$ is computed for each decision D_i, we can compute the MEU as follows:

TABLE VIII
THE CONJUNCTIVE FUSION

V	A	D	Π_{V_1}	Π_{V_2}	Π_{V_3}
T	T	d_1	0	0.225	0.3
T	F	d_1	0	0.0375	0.2
T	T	d_2	0	0.0375	1
T	F	d_2	0	0.0375	0.3
F	T	d_1	0.1	0.375	0
F	F	d_1	1	0.0625	0
F	T	d_2	0.5	0.0625	0
F	F	d_2	1	0.0625	0

TABLE IX
THE DISJUNCTIVE FUSION

V	A	D	$\Pi(V\,\vert A, D)$
T	T	d_1	0.3
T	F	d_1	0.2
T	T	d_2	1
T	F	d_2	0.3
F	T	d_1	0.375
F	F	d_1	1
F	T	d_2	0.5
F	F	d_2	1

$$MEU(D_i, E) = max_{d_{ij}} \sum_{pa'(V)} \Pi(D_i, E) \quad (10)$$

Example 4: Suppose now that we receive a certain information saying that the variable C takes the value T. Since the obtained possibilistic network presented in figure 2 is a multiply connected DAG, so the possibilistic adaptation of junction trees propagation [1] is applied to compute $\Pi(A\,|D, C = T)$ as presented in table X.

TABLE X
THE COMPUTATION OF $\Pi(A\,|D, C = T)$

A	D	C	$\Pi(A\,\vert D, C = T)$
T	d_1	T	1
F	d_1	T	0.2
T	d_2	T	1
F	d_2	T	0.19

After the application of equation (9), we will have $MEU = 0.81$. Thus, the optimal decision is $D^* = d_1$

VI. CONCLUSION

In this paper we have proposed a new extension of possibilistic influence diagrams where several source of information regarding value nodes are available.

Indeed, dependencies between chance nodes are quantified using possibility distributions. Then, an initial expert should define for each value node a set of possible numerical utilities and possibility distributions relative to each consequence and each utility. Then, several assistant experts, characterized by confidence degrees provided by the initial expert, will express their uncertainty concerning value nodes in the same way of the initial expert. In fact, they will provide possibility distributions relative to each consequence and each utility.

To evaluate these *multi-souce possibilistic influence diagrams*, we have proposed an indirect evaluation method based on a processing phase of possibility distributions and on the evaluation algorithm already proposed in [8].

The proposed approach, has been implemented in a *Possibilistic Influence Diagram Toolbox (PIDT)* which can be seen as a decision support system.

A direct improvement of our proposal concerns multi-source possibilistic IDs with several value nodes to deal with multi objective decision problems when uncertainty is modeled in a possibilistic setting.

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