
Volume 195, Issue 1, 16 May 2009
ISSN 0377-2217
ELSEVIER
EUROPEAN JOURNAL OF
OPERATIONAL RESEARCH
Available online at
ScienceDirect
www.sciencedirect.com
www.elsevier.com/locate/ejor

Available online at www.sciencedirect.com
ScienceDirect
European Journal of Operational Research 195 (2009) 223–238
EUROPEAN JOURNAL OF OPERATIONAL RESEARCH
www.elsevier.com/locate/ejor

Decision Support

Qualitative possibilistic influence diagrams based on qualitative possibilistic utilities

Wided Guezguez *, Nahla Ben Amor, Khaled Mellouli

LARODEC, Institut Superieur de Gestion de Tunis, 41 Avenue de la Liberte, Le Bardo 2000, Tunisia

Received 28 December 2006; accepted 28 January 2008
Available online 15 February 2008

Abstract

This paper proposes a new approach for decision making under uncertainty based on influence diagrams and possibility theory. The so-called qualitative possibilistic influence diagrams extend standard influence diagrams in order to avoid difficulties attached to the specification of both probability distributions relative to chance nodes and utilities relative to value nodes. In fact, generally, it is easier for experts to quantify dependencies between chance nodes qualitatively via possibility distributions and to provide a preferential relation between different consequences. In such a case, the possibility theory offers a suitable modeling framework. Different combinations of the quantification between chance and utility nodes offer several kinds of possibilistic influence diagrams. This paper focuses on qualitative ones and proposes an indirect evaluation method based on their transformation into possibilistic networks. The proposed approach is implemented via a possibilistic influence diagram toolbox (PIDT).

Keywords: Decision theory; Influence diagrams; Possibility theory; Ordinal utilities; Binary qualitative utilities

1. Introduction

Decision tools as dynamic programming, simulation techniques and graphical decision models are important techniques to assist decision makers in their task. In this paper, we are interested in graphical decision models which provide a compact and a simple representation of decision problems under uncertainty (Shenoy, 1994).

Within existing graphical models, we are, in particular, interested in influence diagrams (IDs), initially proposed by Howard and Matheson (1984). These networks are a popular framework representing a decision maker's belief and preferences about a sequence of decisions to be made under uncertainty. Indeed, their evaluation generates optimal decisions while maximizing the decision maker's expected utilities. In the standard case, we distinguish direct evaluation methods operating on the original structure (Shachter, 1986; Tatman and Shachter, 1990) and indirect ones (Cooper, 1988; Sanchez and Druzdzel, 2004; Shachter and Poet, 1992; Zhang, 1998; Xiang and Ye, 2001) based on its transformation into a secondary one which will be used in different computations. Most of these latter are based on the transformation of influence diagrams into Bayesian networks (Pearl, 1988; Jensen, 1996).

The quantification of influence diagrams can be done by experts, in such a case they express their uncertainty relative to variables by probability distributions and their preferences through utilities. Nevertheless, in most real problems it is not obvious to provide exact probability distributions and it is easier to express uncertainty qualitatively by ranking different states of the world.

* Corresponding author. Tel.: +216 98477207.
E-mail addresses: widedguezguez@gmail.fr (W. Guezguez), nahla.benamor@gmx.fr (N. Ben Amor), khaled.mellouli@ihec.rnu.tn (K. Mellouli).

0377-2217/$ - see front matter
doi:10.1016/j.ejor.2008.01.051

Moreover, decision makers may encounter several difficulties when expressing their utilities and it may be more flexible to allow them providing a preferential relation between different consequences rather than exact numerical values.

In such situations, standard influence diagrams cannot be applied, thus, our idea is to extend them using a non-classical theory of uncertainty for specifying their numerical component. Namely, we have opted to use possibility theory, initially proposed by Zadeh (1978) and developed by Dubois and Prade (1988) since it offers a natural and simple framework to handle imperfect data.

Few works exist on possibilistic networks and existing ones concern reasoning under uncertainty without considering the decision aspect (Borgelt, 1998; Ben Amor et al., 2001; Fonck, 1994).

Recently, Garcia and Sabbadin (2006) have proposed possibilistic influence diagrams using optimistic and pessimistic utilities (Dubois et al., 1998) for the quantification of value nodes. Nevertheless, Giang and Shenoy (2005) noted that this utility framework is based on axioms relative to uncertainty attitude contrary to the VNM axiomatic system (Neumann and Morgenstern, 1948) based on risk attitude, which does not make a sense in the possibilistic framework since it represents uncertainty rather than risk. Moreover, to use pessimistic and optimistic utilities, the decision maker should classify himself as either pessimistic or optimistic which is not always obvious. To overcome these limitations, Giang and Shenoy (2005) propose a more generalized framework based on the axiomatic system of *possibilistic binary utility* which will be used in this paper.

In addition, the evaluation of possibilistic influence diagrams, proposed in Garcia and Sabbadin (2006), is based on an indirect evaluation method which transforms them into decision trees. Such evaluation method was not successful in the probabilistic framework since, contrary to those based on Bayesian networks, it does not use independencies encoded by influence diagrams to save some computations since decision trees are not able to represent independencies (Zhang et al., 1994). This argument remains available in the possibilistic framework, as it only concerns the graphical component which is the same in the two frameworks.

Possibility theory can be interpreted in two ways: qualitatively, if the handled values reflect only an ordering between the different states of the world or quantitatively, if these values make a sense in the ranking scale. These two interpretations lead to different ways to quantify chance and value nodes, this explains that there exists several possibilistic influence diagrams. In this paper, we will focus on qualitative ones, i.e. those where dependencies between chance nodes are expressed via qualitative possibility distributions and value nodes are quantified using ordinal utility (Neumann and Morgenstern, 1948) or qualitative possibilistic utilities (Giang and Shenoy, 2005).

Our new approach, the so-called *qualitative possibilistic influence diagrams*, benefits from the simplicity and efficiency of standard influence diagrams and from the suitability of possibility theory for modeling qualitative uncertainty.

The success of indirect methods in the standard framework, has motivated us to propose an indirect method to evaluate these models. More precisely, the proposed evaluation method is based on the transformation of qualitative possibilistic influence diagrams into qualitative possibilistic networks (Ben Amor et al., 2001) and on making inference in this secondary structure using the appropriate propagation algorithms.

This paper is organized as follows: Section 2 provides a brief description of the basics of influence diagrams. The necessary background on possibility theory is recalled in Section 3. Section 4 presents possibilistic influence diagrams. Section 5 focuses on qualitative possibilistic influence diagrams. Section 6 proposes an indirect evaluation method relative to this new decision model. Finally, Section 7 briefly presents the possibilistic influence diagram toolbox (PIDT).

2. Influence diagrams

Influence diagrams (IDs) (Howard and Matheson, 1984) are a popular framework representing a decision maker's belief and preferences about a sequence of decisions to be made under uncertainty (Howard and Matheson, 1984). After specifying the structure of an ID, we should define a priori and conditional probability distributions relative to chance nodes. In addition, decision makers should quantify value nodes to express their utilities. Influence diagrams have two components:

1. *Graphical component* (or qualitative component) is a directed acyclic graph (DAG) denoted by $G = (N, A)$, where A is the set of arcs in the graph and N its node set. We can distinguish two kinds of DAGs:
   (a) *Singly connected DAGs:* which contain no loops (i.e. *undirected* cycle).
   (b) *Multiply connected DAGs:* which can contain loops.
   The node set N is partitioned into subsets C, D and V such that:
   - $C = \{C_1, \dots, C_n\}$ is a set of chance nodes which represent relevant uncertain factors for decision problem. Chance nodes are represented by circles.
   - $D = \{D_1, \dots, D_m\}$ is a set of decision nodes which depict decision options, they have a temporal order. Decision nodes are represented by rectangles.
   - $V = \{V_1, \dots, V_k\}$ is a set of value nodes which represent utilities to be maximized, they are represented by lozenges. In what follows, we will handle IDs with a unique value node, for the sake of simplicity we will denote this value node by V.

In what follows c_{ij} (resp. d_{ij}, v_{ij}) denotes the *j*th instance of the variable C_i (resp. D_i, V_i), $Pa(N_i)$ denotes the set of parents of a node $N_i \in N$ and $pa(N_i)$ is an instance of $Pa(N_i)$ and Ω_{N_i} denotes the set of possible values of the node N_i. Arcs in A have different meanings according to their targets. We can distinguish:

- *Conditional arcs* (into chance and value nodes), those that have as target chance nodes represent probabilistic dependence.
- *Informational arcs* (into decision nodes) which imply time precedence.

Influence diagrams are required to satisfy some constraints (Shachter, 1986) to be *regular*, namely the directed graph should not contain cycles, value nodes cannot have children and there exists a directed path that contains all decision nodes. As a result of this property, influence diagrams satisfy the *no-forgetting* property, in the sense that any decision node and its parents correspond to the parent set of all subsequent decision nodes (Zhang, 1998). In our work, we are interesting in regular influence diagrams that satisfy the no-forgetting constraint.

2. *Numerical component* (or quantitative component) consists in evaluating different links in the graph. Namely, each conditional arc which has as target a chance node C_i is quantified by a conditional probability distribution of C_i in the context of its parents denoted by $Pa(C_i)$. Such conditional probabilities should respect the following normalization constraints:
   - If $Pa(C_i) = \emptyset$ (C_i is a root) then the a priori probability relative to C_i should satisfy:
   $$\sum_{c_{ij}\in\Omega_{C_i}} P(c_{ij}) = 1. \quad (1)$$
   - If $Pa(C_i) \neq \emptyset$, then the relative conditional probability relative to C_i in the context of its parents $Pa(C_i)$ should satisfy:
   $$\sum_{c_{ij}\in\Omega_{C_i}} P(c_{ij}|Pa(C_i)) = 1. \quad (2)$$

Each decision alternative may have several consequences according to uncertain variables. The set of consequences is characterized by a utility function. In IDs, consequences are represented by different combinations of value node's parents. Hence, each value node is quantified by a utility function, denoted by U, in the context of its parents.

The quantification of value nodes, in standard IDs, have been made using two ways:

- *Cardinal utility* when decision makers are able to give cardinal numbers representing their satisfaction.
- *Ordinal utility* when decision makers can only define a total preference's relation representing their satisfaction (Neumann and Morgenstern, 1948). This preference's relation must satisfy some axioms (Neumann and Morgenstern, 1948), the most important of them are completeness, reflexivity and transitivity axioms.

Example 1. Let us state a simple decision problem represented by an influence diagram as represented in Fig. 1. It contains 3 chance nodes (A, B, C), 1 decision node (D) and 1 value node (**V**).

Table 1 represents a priori and conditional probabilities for chance nodes A, B and C. Table 2 represents the utility function (ordinal utilities) for the value node V, in the context of its parents (A and D).

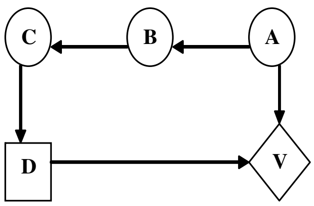

Fig. 1. An example of influence diagram.

Table 1
A priori and conditional possibility distributions for chance nodes

A	$P(A)$	A	B	$P(B\|A)$	B	C	$P(C\|B)$
T	0.4	T	T	0.1	T	T	0.5
F	0.6	F	T	0.2	F	T	0.25
		T	F	0.9	T	F	0.5
		F	F	0.8	F	F	0.75

Table 2
Utility function

A	D	$U(A,D)$
F	F	8
F	T	2
T	F	4
T	T	10

Decision nodes act differently from chance nodes, thus it is meaningless to specify prior probability distribution on them. Moreover, it has no meaning to attach a probability distribution to children nodes of a decision node D_i unless a decision d_{ij} has been taken.

Jensen (2002) has shown that the chain rule relative to influence diagrams can be deduced from the one relative to Bayesian network (Kim and Pearl, 1987) as follows:

$$P(C|D) = \Pi_{C_i \in C} P(C_i|Pa(C_i)) \tag{3}$$

Evaluation of influence diagrams: Once the ID is constructed, it can be used to identify the optimal policy, this can be ensured via evaluation algorithms which allow to generate the best strategy yielding to the highest expected utility.

Within proposed evaluation algorithms, we can distinguish direct methods (Shachter, 1986; Tatman and Shachter, 1990) which operate directly on influence diagrams or indirect methods (Cooper, 1988; Sanchez and Druzdzel, 2004; Shachter and Poet, 1992; Zhang, 1998; Xiang and Ye, 2001) which transform them into a secondary structure and then evaluate these structures.

Howard and Matheson (1984) have defined an indirect evaluation method, they used decision trees as a secondary structure to determine the optimal policy. Then, in 1986, Shachter proposed a direct evaluation method based on arc reversal and node deletion. Since 1988, several indirect evaluation methods, which reduce the influence diagram evaluation problem into a Bayesian network inference one, have been developed.

The use of decision trees for evaluation influence diagrams does not use conditional independencies and direct evaluation requires a lot of probabilistic calculations which justify the great development of indirect methods initiated by Cooper (1988) for the particular case of influence diagrams with a unique value node.

The key idea of indirect methods using Bayesian network as a secondary structure is to transform decision and value nodes into chance nodes to obtain a Bayesian network. In this work, we will only focus on these indirect methods and especially on the one proposed by Cooper (1988).

3. Background of possibility theory

Possibility theory was initially proposed by Zadeh (1978) and was developed by Dubois and Prade (1988). This section briefly recalls basic elements of possibility theory, for more details see (Dubois and Prade, 1988).

3.1. Basic elements

The basic buildings block in the possibility theory is the notion of possibility distribution denoted by π, it is a mapping from the universe of discourse denoted by $\Omega = \{\omega_1, \ldots, \omega_n\}$ to the unit interval $[0,1]$.

This scale has two interpretations, a quantitative one when the handled values have a real sense and a qualitative one when the handled values reflect only an ordering between the different states of the world (Dubois et al., 2001). In the first case, the *product operator* can be applied while in the second one, the *min operator* is used.

A possibility degree is the value from the interval $[0,1]$ associated to each element ω of Ω. The possibility measure of any subset $\psi \subseteq \Omega$ is defined as follows:

$$\Pi(\psi) = \max_{\omega \in \psi} \pi(\omega). \tag{4}$$

A possibility distribution is said to be normalized, if $\max_{\omega \in \psi} \pi(\omega) = 1$.

In the possibilistic framework, extreme forms of partial knowledge can be represented as follows:

- *Complete knowledge*: $\exists \omega_i \in \Omega,\ \text{s.t}\pi(\omega_i) = 1$ and $\omega_j \neq \omega_i,\ \pi(\omega_j) = 0$.
- *Total ignorance*: $\forall \omega_i \in \Omega,\ \pi(\omega_i) = 1$.

The two interpretations of the possibilistic scale induce two definitions of the conditioning:

- Min-based conditioning relative to the ordinal setting:

$$\pi(\omega|_m\psi) = \begin{cases} 1 & \text{if } \pi(\omega) = \Pi(\psi) \text{ and } \omega \in \psi, \\ \pi(\omega) & \text{if } \pi(\omega) < \Pi(\psi) \text{ and } \omega \in \psi, \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

- Product-based conditioning relative to the numerical setting:

$$\pi(\omega|_p\psi) = \begin{cases} \frac{\pi(\omega)}{\Pi(\omega)} & \text{if } \omega \in \psi, \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

3.2. Qualitative possibilistic utility

Classical approaches for decision making under uncertainty are based on maximal expected utility. However, the computation of this measure is not always obvious especially in complex decision problems when probabilities and numerical utilities are difficult to assess.

To overcome this limitation many researches aim to introduce non-probabilistic decision theory. Indeed, many efforts have been made to create an axiomatic basis for possibility-based decision theory. We can distinguish those following Von Neumann and Morgenstern axiomatic system (Neumann and Morgenstern, 1948) and those following Savage axiomatic system (Savage, 1972). More precisely, Dubois et al. (1998) proposed pessimistic and optimistic qualitative utilities and Giang and Shenoy (2005) introduced the concept of binary qualitative utility as a possibilistic counterpart to VNM's axiomatic system. In our work, we are only interested in possibilistic qualitative utility following VNM's axiomatic system.

Two qualitative criteria that evaluate the worth of decision have been defined in the literature. It concerns pessimistic (Whalen, 1984) and optimistic (Yager, 1979) criteria based on possibility events. Decision makers are supposed supplying an order between different consequences expressing their attitude in front of risk. A decision maker is said to be neutral if he is neither pessimistic nor optimistic.

Dubois et al. (1998) defined qualitative possibilistic decision theory based on the uncertainty-aversion decision maker's behavior (a pessimistic and an optimistic view).

Giang and Shenoy (2005) criticized pessimistic and optimistic utilities presented by Dubois et al. (1998), their arguments are based on the fact that proposed frameworks for possibilistic utilities are based on axioms relative to uncertainty attitude contrary to the VNM axiomatic system based on risk attitude which does not make a sense in the possibilistic framework since it represents uncertainty rather than risk. Moreover, to use pessimistic and optimistic utilities, the decision maker should classify himself as either pessimistic or optimistic which is not always obvious.

Giang and Shenoy (2005) investigated a new axiomatic system for preference relation for the set of qualitative lotteries which are the possibilistic counterpart of probabilistic lotteries (probability distribution over the set of consequences). This axiomatic system is based on the fact that lotteries that realize in the best prize or in the worst prize (called canonical lotteries) play an important role in decision making.

To represent a canonical lottery in the probabilistic case, we only need a simple number which is the probability p of getting the best prize. For the worst case, this probability is obviously equal to $1 - p$.

In possibility theory, we need two measures to represent a canonical lottery since the knowledge that the best prize is fully possible could imply nothing about the possibility of getting the worst prize. The concept of binary utility was introduced to represent canonical lotteries in the possibility theory (Giang and Shenoy, 2005). In what follows, we present basic elements of this concept (for more details see (Giang and Shenoy, 2005)).

- Let X be a finite set of consequences (i.e. outcomes or prizes). In the set X, we can distinguish two particular elements namely the *best* and the *worst* consequence.
- Let Π_X be a set of canonical lotteries.
- Let W be a linearly ordered scale and U_W be a set of pairs of elements in W such that one of them is equal to 1. Indeed, the set U_W equipped with the order $\succeq$ is a binary utility scale.
- Let V be a finite ordered scale representing uncertainty, such that $\forall v \in V,\ 0 \leqslant v \leqslant 1$.

Giang and Shenoy defined a *basic utility assessment* as a function $BU : X \to U_W$ that assigns an utility for each prize in X, such that $\forall x \in X, BU(x) = \langle BU^L(x), BU^R(x)\rangle$, where $BU^R(x)$ and $BU^L(x)$ are respectively, the right and the left element of the pair $BU(x)$.

Note that $BU^R(x)$ represents the utility affected to the worst prize and $BU^L(x)$ represents the one affected to the best prize.

A possibilistic utility function PU relative to each lottery π is a mapping from Π_X to U_W defined as follows:

$$PU(\pi) = \max_{x \in X} \min(k(\pi(x)), BU(x)), \tag{7}$$

where k is a binary vector function defined as follows:

Definition 1. Let $k = \langle k_1, k_2 \rangle$ such that $\forall r = 1, 2, k_r$ is an order preserving mapping from V to W, such that $k_r(1) = 1$ and $k_r(0) = 0$. The binary vector function k is relative to the type of the decision maker (i.e. optimistic, pessimistic, neutral). More precisely:

- If the DM is pessimistic then $k_1(0) = 0, \forall v \in \{V - 0\} k_1(v) = 1$ and $\forall v \in V k_2(v) = v$.
- If the DM is optimistic then $\forall v \in V k_1(v) = v, k_2(0) = 0$ and $\forall v \in \{V - 0\} k_2(v) = 1$.
- If the DM is neutral then $\forall v \in V k_1(v) = k_2(v) = v$.

Example 2. Let us consider the uncertainty scale $V = \{1, 0.7, 0.5, 0\}$. Binary utilities for the three types of decision maker are given by Table 3.

The possibilistic utility of the lottery π denoted by $PU(\pi)$ is considered as the *expected qualitative utility* of π. This means that the expected qualitative utility of the consequence x is $PU(x)$, since each consequence can be written in the form of a lottery π_x.

4. Possibilistic influence diagrams

Possibilistic influence diagrams are a possibilistic adaptation of standard influence diagrams, as the latter they have two components:

1. *A graphical component* defined by a directed acyclic graph (DAG), denoted by $G(N, A)$, where N is the set of chance, decision and value nodes and A is the set of arcs in the directed graph.
2. *A numerical component* evaluating different dependencies between chance nodes and utilities for value nodes.
   - For each chance node C_i, we should provide conditional possibility degree $\Pi(c_{ij}|pa(C_i))$ of each instance c_{ij} of C_i in the context of each instance of its parents. In order to satisfy the normalization constraint, these conditional distributions should satisfy:

     $$\max_{c_{ij}} \Pi(c_{ij}|_{\diamondsuit} pa(C_i)) = 1, \quad \forall pa(C_i), \tag{8}$$

     where $\diamondsuit$ is the min-based or the product-based conditioning according to the interpretation of the possibilistic scale. Note that for root chance nodes, i.e. $(Pa(C_i) = \emptyset)$, (8) corresponds to $\max_{c_{ij}} \Pi(c_{ij}) = 1$.
   - For each value node V_i, there are three ways to represent decision maker's preferences on the set of consequences, using cardinal, ordinal or qualitative binary utilities. Note that likewise standard influence diagrams, decision nodes in possibilistic IDs are not quantified.

Table 3
Binary utilities for the three types of DM

v	Pessimistic DM, $\langle k_1(v), k_2(v) \rangle$	Optimistic DM, $\langle k_1(v), k_2(v) \rangle$	Neutral DM, $\langle k_1(v), k_2(v) \rangle$
1	$\langle 1, 1 \rangle$	$\langle 1, 1 \rangle$	$\langle 1, 1 \rangle$
0.7	$\langle 1, 0.7 \rangle$	$\langle 0.7, 1 \rangle$	$\langle 0.7, 0.7 \rangle$
0.5	$\langle 1, 0.5 \rangle$	$\langle 0.5, 1 \rangle$	$\langle 0.5, 0.5 \rangle$
0	$\langle 0, 0 \rangle$	$\langle 0, 0 \rangle$	$\langle 0, 0 \rangle$

Table 4
Different variants of possibilistic influence diagrams

Value nodes chance nodes	Cardinal utility	Ordinal utility	Qualitative possibilistic utility (ΠO)
Product-based conditioning (*)	$\Pi\mathrm{ID}_*^{\mathrm{C}}$	$\Pi\mathrm{ID}_*^{\mathrm{O}}$	$\Pi\mathrm{ID}_*^{\Pi\mathrm{O}}$
Min-based conditioning (min)	$\Pi\mathrm{ID}_{\min}^{\mathrm{C}}$	$\Pi\mathrm{ID}_{\min}^{\mathrm{O}}$	$\Pi\mathrm{ID}_{\min}^{\Pi\mathrm{O}}$

Different combinations of the quantification of chance and utility nodes offer several kinds of possibilistic influence diagrams summarized in Table 4.

These different kinds of possibilistic IDs can be regrouped into three principal classes:

- *Product-based possibilistic influence diagrams* where both dependencies between chance nodes and value nodes are quantified in a genuine numerical setting (i.e. $\Pi\mathrm{ID}_{*}^{\mathrm{C}}$).
- *Min-based possibilistic influence diagrams or qualitative possibilistic ID* where both dependencies between chance nodes and value nodes are quantified in a qualitative setting used for encoding an ordering between different states of the world (i.e. $\Pi\mathrm{ID}_{\min}^{\mathrm{O}}$ and $\Pi\mathrm{ID}_{\min}^{\Pi O}$).
- *Mixed possibilistic influence diagrams* where dependencies between chance nodes and value nodes are not quantified in the same setting (i.e. $\Pi\mathrm{ID}_{\min}^{\mathrm{C}}$, $\Pi\mathrm{ID}_{*}^{\mathrm{O}}$ and $\Pi\mathrm{ID}_{*}^{\Pi O}$).

In a previous study (Guezguez et al., 2006), we have shown that product-based possibilistic influence diagrams are very close to standard ones since they share same operators (i.e. product operator) and same features. This is not the case for min-based possibilistic influence diagrams since the minimum operator has different properties from the product one. This explains our interest to this kind of influence diagrams in this paper.

5. Qualitative possibilistic influence diagrams

In these qualitative models, conditionals between chance nodes are defined using min-based conditioning expressed by (5). This means that $\diamondsuit$ in (8) corresponds to this type of conditioning.

There are two ways to quantify value nodes in min-based possibilistic influence diagrams:

1. *Qualitative possibilistic IDs with ordinal utilities (denoted by $\Pi ID_{\min}^{\mathrm{O}}$)*: in this case, the decision maker should provide a complete preferential relation, denoted by $\succeq$, between different consequences.
2. *Qualitative possibilistic IDs with binary utilities (denoted by $\Pi ID_{\min}^{\Pi O}$)*: in that case, the decision maker should provide a complete preferential relation, denoted by $\succeq$, between different consequences and a utility scale W. Obviously, he should also precise his behavior (i.e. pessimistic, optimistic or neutral).

To define the chain rule for the two kinds of qualitative possibilistic influence diagrams (i.e. $\Pi\mathrm{ID}_{\min}^{\mathrm{O}}$ and $\Pi\mathrm{ID}_{\min}^{\Pi O}$), our idea is to proceed in the same way than the standard influence diagrams by ignoring utility nodes and links into decision nodes.

Since decision nodes are not quantified, they act differently from chance nodes, thus for a given chance node C_i and a decision node D_i, it is meaningless to consider $\Pi(c_{ij}, d_{ij})$. In fact, what is meaningful is $\Pi(c_{ij}|_m do(d_{ij}))$, where $do(d_{ij})$ is the particular operator defined by Pearl. When iterating this reasoning we can bunch the whole decision nodes together and express the joint possibility distribution of different chance nodes conditioned by decision nodes. This means that if we fix a particular configuration of decision nodes, say d, we get a qualitative possibilistic network (Ben Amor et al., 2001) representing $\Pi(C|_m do(d))$, i.e. the joint possibility relative to C, in the context of d.

Using the chain rule relative to qualitative possibilistic networks (Ben Amor et al., 2001), the following chain rule for qualitative possibilistic IDs can be inferred:

$$\pi_m(C|_m D) = \min_{C_i \in C} \Pi(C_i|_m Pa(C_i)). \tag{9}$$

In other words, possibilistic influence diagrams are a compact representation of the joint distribution relative to chance nodes conditioned by a fixed configuration of decision nodes.

The following example illustrates the two kinds of qualitative possibilistic influence diagrams.

Example 3. Let us reconsider the possibilistic ID represented in *Fig. 1*. Possibility distributions for chance nodes A, B and C are represented in Table 5. For the quantification of the node V, we can consider two decision makers who express their utilities in an ordinal manner but in different ways:

Table 5
A priori and conditional possibility distributions for chance nodes

A	$\Pi(A)$	A	B	$\Pi(B|A)$	B	C	$\Pi(C|B)$
T	1	T	T	0.9	T	T	1
F	0.6	F	T	0.2	F	T	0.3
		T	F	1	T	F	0.2
		F	F	1	F	F	1

- the first expresses his utility via a preferential relation $\succeq$ between different consequences, as follows:

  $(D = Act2 \wedge A = F) \succeq (D = Act1 \wedge A = T) \succeq (D = Act1 \wedge A = F) \succeq (D = Act2 \wedge A = T)$.

- the second, who is optimistic, uses binary utilities, by providing a preferential relation $\succeq$ and an utility scale W as follows:

  $(D = Act1 \wedge A = T) \succeq (D = Act2 \wedge A = F) \succeq (D = Act2 \wedge A = T) \succeq (D = Act1 \wedge A = F)$

  $W = \{1, 0.7, 0.5, 0\}$.

Note that, the DM should precise his behavior towards the risk since, $\succeq$ and W are not sufficient to have this information.

It is clear that this induces two different possibilistic influence diagrams, the first one will be referred to by $\Pi ID1$ and the second by $\Pi ID2$.

6. Evaluation of qualitative possibilistic influence diagrams

Given a possibilistic ID, we should evaluate it in order to generate optimal decisions. As we have seen in Section 2, there are two approaches to evaluate standard influence diagrams, namely, direct and indirect ones.

Direct evaluation methods (Shachter, 1986; Tatman and Shachter, 1990) require heavy computations since they are based on arc reversal and node deletion, contrary to indirect ones which are based on the transformation of influence diagrams into Bayesian networks. This explains the great development of indirect methods in the probabilistic case (Cooper, 1988; Sanchez and Druzdzel, 2004; Shachter and Poet, 1992; Zhang, 1998; Xiang and Ye, 2001).

The success of indirect evaluation methods for standard IDS, motivates us to develop an indirect evaluation method for qualitative possibilistic influence diagrams. Our choice is reinforced by the fact that a possibilistic counterpart of Bayesian networks has been recently developed as well as their propagation algorithms (Ben Amor et al., 2001).

More precisely, we will develop a possibilistic counterpart of Cooper's method (1988) for the particular case of influence diagram with a unique value node, since it represents the basis of existing indirect methods.

Thus, the principle of our evaluation algorithm is to transform decision and value nodes into chance nodes in order to obtain a qualitative possibilistic network, and then to use this secondary structure to compute maximal expected utilities via a propagation process. These two major phases are detailed in what follows:

6.1. Transformation phase

This phase consists in transforming decision and value nodes into chance nodes.

6.1.1. Decision nodes transformation

Each decision node D_i in the possibilistic influence diagram is transformed into a chance node which should be quantified. In the probabilistic case, this quantification is ensured by an equi-probable distribution. Nevertheless, this is not really appropriate, since equi-probability represents randomness rather than total ignorance. This problem can be overcome in the possibilistic framework where our ignorance about the new chance node can be suitably represented via a uniform possibility distribution. More formally:

$$\Pi(d_{ij}|_m pa(D_i)) = 1, \quad \forall d_{ij}, pa(D_i). \tag{10}$$

6.1.2. Value node transformation

The value node V will be converted into a new binary chance node having two values, i.e. False (F) and True (T). To quantify this node, we should compute its possibility distribution $\Pi(V|_m Pa(V))$. This quantification depends on the kind of utility used in specifying decision maker's preferences. In what follows, we distinguish the case of ordinal and binary utilities.

- *Case of ordinal utilities*: in this case, the value node is transformed into a new chance node V characterized by a possibility distribution derived from the original preferential relation $\succeq$ between different consequences. The principle of this transformation, is to first convert $\succeq$ into numerical utilities as shown by Proposition 1, then to use these utilities to derive the new possibility distribution.

Proposition 1. *Let $\succeq$ be a preferential relation between different consequences. Then, the order induced by $\succeq$ can be rescaled into a numerical scale as follows*:

$$U(pa(V)) = \mathrm{card}(Pa(V)) - \mathrm{rank}(pa(V)) + 1. \tag{11}$$

where $card(Pa(V))$ is a function that determines the number of combinations of $pa(V) \in Pa(V)$ and the function $rank(pa(V))$ is the rank of $pa(V)$ in the preference relation $\succeq$. More precisely, it is a mapping from the $\succeq$ to N such that the most preferred consequence in $\succeq$ has as rank 1 and $\forall C_i, rank(C_{i+1}) = rank(C_i) + 1$.

Proof 1. We will prove that the proposed transformation method verifies the fundamental theorem of VNM (Neumann and Morgenstern, 1948). This theorem denotes that for each preference relation that satisfies the three axioms of orderability, reflexivity and transitivity, there exists a utility function such that for any two consequences A and B, we have:

$$\begin{cases} A \succeq B & \text{iff } U(A) \geqslant U(B), \\ A \sim B & \text{iff } U(A) = U(B). \end{cases}$$

For our case, let $pa_1(V)$ and $pa_2(V)$ be two consequences such that $pa_1(V) \succeq pa_2(V)$, we have:

$$\begin{aligned} & \text{rank}(pa_1(V)) \leqslant \text{rank}(pa_2(V)) \\ & \Rightarrow -\text{rank}(pa_1(V)) + 1 \geqslant -\text{rank}(pa_2(V)) + 1 \\ & \Rightarrow \text{card}(Pa(V)) - \text{rank}(pa_1(V)) + 1 \geqslant \text{card}(Pa(V)) - \text{rank}(pa_2(V)) + 1. \end{aligned}$$

Thus, $U(pa_1(V)) \geqslant U(pa_2(V))$. □

Once the set of utilities assigned to different consequences is computed, the possibility distribution attached to the new chance node can be computed as follows:

$$\Pi(v = T|_m pa(V)) = \frac{U(pa(V)) - U_{\min}}{U_{\max} - U_{\min}}, \tag{12}$$

$$\Pi(v = F|_m pa(V)) = \frac{U(pa(V)) - U_{\max}}{U_{\min} - U_{\max}}, \tag{13}$$

where $U_{\max}$ (resp. $U_{\min}$) is the maximal utility level (resp. minimal utility level).

The obtained qualitative possibility distribution $\Pi(V|Pa(V))$ may be sub-normalized. Thus, we should normalize it in order to satisfy Eq. (8). For the sake of clarity, we will denote the normalized distribution by Π_N. The normalization can be insured as follows: $\forall pa(V) \in Pa(V), \forall v \in \{T, F\}$,

$$\Pi_N(v|_m pa(V)) = \begin{cases} 1 & \text{if } max(\Pi(v|_m pa(V)), \Pi(\neg v|_m pa(V))) = \Pi(v|_m pa(V)), \\ \Pi(v|_m pa(V)) & \text{otherwise}, \end{cases} \tag{14}$$

- *Case of binary utilities*: in this case, the decision maker provides his utility scale and the preferential relation on the set of consequences and defines his behavior. These values will be used to quantify the new chance node V by computing $\Pi(V|Pa(V))$ as follows:
  1. The binary utility scale U_W is derived from the utility scale W provided by the decision maker.
  2. According to the decision maker behavior and his preference relation, the binary utilities are assigned to each consequence.
  3. In order to transform binary utilities into $\Pi(V|Pa(V))$, we propose to define the reciprocal function of the binary vector function k (e.g. Section 3.2), denoted by k^{-1} as follows:

Proposition 2. *Let k^{-1} be the reciprocal function of the binary vector function k. k^{-1} is a mapping from U_W to W, and it is defined independently of the decision maker behavior. More formally*, $\forall w \in W$:

$$k^{-1}(BU(w)) = \min(BU^R(w), BU^L(w)), \tag{15}$$

where $BU^R(w)$ and $BU^L(w)$ are respectively, the right and the left element of the binary utility.

Proof 2. We will prove that whatever the behavior of the decision maker is, we have $\forall w \in W, k^{-1}(k(w)) = w$.

–if the DM is *pessimistic* then $k^{-1}(k(0)) = k^{-1}(\langle 0,0\rangle) = 0$, and $k^{-1}(k(w)) = k^{-1}(\langle 1,w\rangle) = w$,

–if the DM is *optimistic* then $k^{-1}(k(0)) = k^{-1}(\langle 0,0\rangle) = 0$, and $k^{-1}(k(w)) = k^{-1}(\langle w,1\rangle) = w$,

–if the DM is *neutral* then $k^{-1}(k(w)) = k^{-1}(\langle w,w\rangle) = w$. □

The possibility distribution $\Pi(v = T|Pa(V))$ resulted from Eq. (15) may be sub-normalized. Thus, we should normalize it in order to satisfy Eq. (8). To satisfy this constraint, we should first compute $\Pi(v = F|Pa(V))$. However, the computation of the exact value of this degree is not possible from available data since we are in an ordinal setting. Thus, our idea is to compute it using (13), then to normalize the distribution issued from (15) using (14).

6.2. Propagation phase

The possibilistic network issued from the transformation phase can be used to generate optimal decisions by computing the maximal expected utility (MEU) relative to each decision node. This computation is ensured by applying a propagation process.

Since we are in a qualitative setting, we should select a min-based propagation algorithms for qualitative possibilistic networks (Fonck, 1994; Borgelt, 1998; Ben Amor et al., 2001) according to the DAG structure (singly or multiply connected) and to its size. Note that the propagation process is an NP-complete problem.

We can distinguish two exact min-based propagation algorithms defined according to the nature of the DAG. Namely, the qualitative possibilistic adaptation of the centralized version of Pearl's algorithm Pearl (1986) is used when the DAG is singly connected and the qualitative possibilistic adaptation of junction trees propagation algorithm (Ben Amor, 2002) is appropriate for multiply connected DAGs. If these two algorithms are blocked, an approximate approach, such as the anytime algorithm (Ben Amor et al., 2001), can be applied.

To illustrate the propagation phase, we briefly recall the min-based propagation algorithm in junction trees.

6.2.1. Propagation in junction trees

The propagation algorithm in junction trees is similar to the one proposed by Jensen for multiply connected Bayesian networks (Jensen, 1996). This algorithm is based on four steps ($S1 - S4$):

6.2.2. Step S1: Building junction tree

The first step consists in transforming the initial network into a secondary structure corresponding to a junction tree, in order to eliminate existing loops. This step is identical to the one proposed for Bayesian networks since it is independent from numerical values. The principle steps to build a junction tree from a DAG can be summarized as follows (Huangn and Darwiche, 1994):

- *Moralization of the initial graph*: This step allows to create an undirected graph from the initial one by connecting the parent set of each node.
- *Triangulation of the moral graph*: The goal of this step is to identify sets of variables which can be regrouped in clusters, denoted by Cl_i. It is possible to have different triangulation of a moral graph. However, it is important to find the optimal triangulation assuring the minimization of cluster sizes to allow local computations. The task of finding an optimal triangulation is stated as an NP-complete problem (Cooper, 1988) and several heuristics have been proposed (Kjaerulff, 1990).
- *Building a junction tree* $\mathscr{JT}$: To build an optimal junction tree, clusters identified in the previous step should be connected such that all cluster pertaining to the path between any two clusters Cl_i and Cl_j should contain $Cl_i \cap Cl_j$. Once adjacent clusters are identified, we should insert between each pair of clusters Cl_i and Cl_j a separator denoted by S_{ij} and containing common variables.

6.2.3. Step S2: Initialization

This step allows the quantification of the $\mathscr{JT}$, built in the previous step, by transforming initial conditional possibility distributions into local joint distributions attached to clusters and separators. More specifically, a potential $\pi^t_{Cl_i}$ (resp. $\pi^t_{S_{ij}}$), where t is relative to the propagation step, is affected to each cluster Cl_i (resp. separator S_{ij}) of $\mathscr{JT}$. In particular, $t = I$ corresponds to the initialization step. The main steps of the initialization procedure are as follows:

- Affect a uniform possibility distribution 1 (i.e. a uniform possibility distribution) where all the elements are equal to 1 for all clusters and separators.
- For each variable N_k, choose a cluster Cl_i containing $\{N_k\} \cup Pa(N_k)$ and modify its local possibility distribution such that $\pi^I_{Cl_i} \leftarrow min(\pi^I_{Cl_i}), \Pi(N_k|Pa(N_k))$.

6.2.4. Step S3: Global propagation

Once the junction tree $\mathscr{JT}$ is initialized, the global propagation is performed in order to make it globally consistent, i.e. the link existing between any two adjacent clusters Cl_i and Cl_j in $\mathscr{JT}$ is coherent. More formally:

$$\max_{Cl_i \setminus S_{ij}} \pi^t_{Cl_i} = \pi^t_{S_{ij}} = \max_{Cl_j \setminus S_{ij}} \pi^t_{Cl_j}, \qquad (16)$$

where $\max_{Cl_i \setminus S_{ij}} \pi^t_{Cl_i}$ is the marginal distribution of S_{ij} defined from $\pi^t_{Cl_i}$.

The global propagation is ensured via a message passing mechanism between clusters which starts by choosing an arbitrary cluster to be a pivot node, then follows two main phases:

- *A collect-evidence* phase in which each cluster passes a message to its adjacent cluster in the pivot direction (in this direction each cluster has a unique adjacent cluster).
- *A distribute-evidence* phase in which each cluster passes a message to its adjacent clusters away from the pivot direction beginning by the pivot itself until reaching the leaves of the graph.

If a cluster Cl_i sends a message to its adjacent cluster Cl_j then the potentials of Cl_i, Cl_j and their separator S_{ij} are updated as follows:

1. Save the same potential for $Cl_i : \pi^{t+1}_{Cl_i} \leftarrow \pi^t_{Cl_i}$.
2. Update the potential of $S_{ij} : \pi^{t+1}_{S_{ij}} \leftarrow \max_{Cl_i \setminus S_{ij}} \pi^t_{Cl_i}$.
3. Update the potential of $Cl_j : \pi^{t+1}_{Cl_j} \leftarrow min(\pi^t_{Cl_j}, \pi^{t+1}_{S_{ij}})$.

6.2.5. Step S4: Marginalization

Let $\pi^{Cl}_{Cl_i}$ be the potential of Cl_i after the global propagation step, then it can be proved that this potential encodes the local joint distribution of Cl_i induced from the initial graph, i.e. $\pi^{Cl}_{Cl_i}$. Thus, the computation of the marginals relative to each variable N_k can be ensured by the marginalization of the potential of any cluster Cl_i containing it, i.e. $\Pi(N_k) = \max_{Cl_i \setminus N_k} \pi^{Cl}_{Cl_i}$.

6.2.6. Handling the evidence

The manipulation of an evidence e, corresponding to the set of instantiated variables, allows the computation for any variable N_k of $\Pi(N_k \wedge e)$ instead of $\Pi(N_k)$.

This can be ensured by integrating this evidence in the junction tree by first encoding the local evidence related to each variable N_k as follows:

$$\Lambda_{N_k}(n_k) = \begin{cases} 1 & \text{if } N_k \text{ is not instantiated,} \\ 1 & \text{if } N_k \text{ is instantiated as } n_k, \\ 0 & \text{if } N_k \text{ is instantiated by a different value of } n_k. \end{cases} \tag{17}$$

Then this local evidence is integrated by choosing a cluster Cl_i containing N_k and updating its potential as follows: $\pi^I_{Cl_i} \leftarrow min(\pi^I_{Cl_i}, \Lambda_{N_k})$.

According to the global propagation step and the hypothesis that we have a certain evidence e, the potential of each cluster Cl_i encodes $\Pi(Cl_i \wedge e)$. Thus, if we marginalize the potential $\pi^{Cl}_{Cl_i}$ of a cluster Cl_i with respect to a variable N_K pertaining to it, we obtain the possibility degree of N_K and the evidence e, i.e. $\Pi(N_k \wedge e) = \max_{Cl_i \setminus N_k} \pi^{Cl}_{Cl_i}$.

However, our goal is to compute $\Pi(N_k|e)$, this value can be easily obtained by applying the min-based conditioning (see Eq. (5)) as follows:

$$\Pi(N_k|e) = \begin{cases} \Pi(N_k \wedge e) & \text{if } \Pi(N_k \wedge e) < \Pi(e) = \max_{N_k} \Pi(N_k \wedge e), \\ 1 & \text{otherwise.} \end{cases} \tag{18}$$

6.2.7. Computation of the maximal expected utility

In order to generate the optimal decision strategy, we should compute the maximal expected utility (MEU) relative to each decision node starting by the last one D_m to the first one D_1. For the node D_i, we should integrate already computed optimal decisions, i.e. those relative to $D_1, \ldots, D_{i-1}$.

This computation depends on the used utility for the quantification of the value node:

- In the case of *ordinal utilities*, the maximal expected utility relative to a decision node D_i is defined as follows:

$$\mathrm{MEU}(D_i, E) = \max_{d_{ij}} [max_{Pa'(V)} min(\Pi(v = T|Pa(V)), \Pi(Pa'(V)|d_{ij}, E))], \tag{19}$$

where $Pa'(V)$ denotes the set of chance nodes in $Pa(V)$.Note that $\Pi(v = T|Pa(V))$ is a transformation of $U(Pa(V))$ into the unit interval $[0, 1]$ using 12 in order to let $U(Pa(V))$ and $\Pi(Pa'(V)|d_{ij}, E)$ commensurable since, the min operator is used. To compute $\Pi(Pa'(V)|d_{ij}, E)$ and $\Pi(v = T|Pa(V))$, we should use the appropriate possibilistic propagation algorithm as explained in the introduction of Section 6.2.

- In the case of *binary utilities*, we will use the fact that the expected qualitative utility of a lottery π corresponds to $PU(\pi)$ (see Section 3.2). Since each consequence x can be written in the form of a lottery, then we can conclude that the expected utility of each consequence $x \in X$ can be computed as follows:

$$\mathrm{EU}(x) = \mathrm{PU}(x) = U(x), \tag{20}$$

Hence, the maximal expected utility relative to a decision node D_i can be computed as follows:

$$\mathrm{MEU}(D_i, E) = \max_{d_{ij}} U(Pa(V)) = max_{d_{ij}} \Pi(v = T|Pa(V), d_{ij}, E). \tag{21}$$

As it is done in the case of ordinal utilities, $\Pi(v = T|Pa(V), d_{ij}, E)$ is computed via the appropriate propagation algorithm (see Section 6.2).

Example 4. Let us consider the qualitative possibilistic influence diagrams $\Pi ID1$ and $\Pi ID1$ given in Example 1. The transformation phase generates the qualitative possibilistic network represented in Fig. 2. Note that the graphical component of this network is the same for the two influence diagrams since they shame the same initial structure (see Fig. 1).

In the same way, the conditional possibility distribution relative to the D (presented in Table 6) is identical for $\Pi ID1$ and $\Pi ID1$ since it represents our ignorance about this node (see Eq. (10)).

Obviously, the quantification of the node V, in context of its parents D and A depends on the initial quantification of the value node. More precisely:

- For $\Pi ID1$, the preferential relation $(D = Act2 \wedge A = F) \succeq (D = Act1 \wedge A = T) \succeq (D = Act1 \wedge A = F) \succeq (D = Act2 \wedge A = T)$ is transformed into numerical utilities using (11) as presented in Table 7. Then, these values are used to compute the conditional possibility distribution of V using ()()()(12)–(14) as presented in Table 8.
- For $\Pi ID2$, the binary utility scale derived from the utility scale $W = \{1, 0.7, 0.5, 0\}$ is defined by: $U_W = \{\langle 0,1\rangle; \langle 0.5,1\rangle; \langle 0.7,1\rangle; \langle 1,1\rangle; \langle 1,0.7\rangle; \langle 1,0.5\rangle; \langle 1,0\rangle\}$. Table 9 for different kinds of decision makers and their transformation using 15. Table 10 presents the new normalized possibility distribution attached to V.

Suppose that we receive a certain information saying that the variable C takes the value T, then the computation of the impact of such information differs from $\Pi ID1$ and $\Pi ID2$. More precisely:

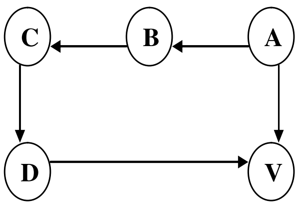

Fig. 2. Resulted possibilistic network.

Table 6
The possibility distribution relative to the decision node D

C	D	$\Pi(D\|C)$
T	*Act1*	1
F	*Act2*	1
T	*Act2*	1
F	*Act1*	1

Table 7
Numerical utilities

A	D	$U(A,D)$
T	*Act1*	2
F	*Act2*	1
T	*Act2*	4
F	*Act1*	3

Table 8
The possibility distribution of the value node V

V	A	D	$\Pi(V\|A,D)$	$\Pi_N(V\|A,D)$
T	T	$Act1$	1/3	1/3
T	F	$Act2$	0	0
T	T	$Act2$	1	1
T	F	$Act1$	2/3	1
F	T	$Act1$	2/3	1
F	F	$Act2$	1	1
F	T	$Act2$	0	0
F	F	$Act1$	1/3	1/3

Table 9
Binary utilities and their transformation using k^{-1}

A	D	Pessimistic DM		Optimistic DM		Neutral DM	
		$U(A,D)$	$k^{-1}(U(A,D))$	$U(A,D)$	$k^{-1}(U(A,D))$	$U(A,D)$	$k^{-1}(U(A,D))$
T	$Act1$	$\langle 0,0\rangle$	0	$\langle 1,1\rangle$	1	$\langle 1,0\rangle$	0
F	$Act2$	$\langle 1,0.5\rangle$	0.5	$\langle 0.7,1\rangle$	0.7	$\langle 1,0.5\rangle$	0.5
T	$Act2$	$\langle 1,0.7\rangle$	0.7	$\langle 0.5,1\rangle$	0.5	$\langle 0.7,1\rangle$	0.7
F	$Act1$	$\langle 1,1\rangle$	1	$\langle 0,1\rangle$	0	$\langle 0,1\rangle$	0

Table 10
New possibility distribution $\Pi(V|A,D)$ (values of $\Pi_N(V|A,D)$ are under brackets)

V	A	D	Pessimistic DM		Optimistic DM		Neutral DM	
T	T	$Act1$	0	(0)	1	(1)	0	(0)
T	F	$Act2$	0.5	(1)	0.7	(1)	0.5	(1)
T	T	$Act2$	0.7	(1)	0.5	(1)	0.7	(1)
T	F	$Act1$	1	(1)	0	(0)	0	(0)
F	T	$Act1$	1	(1)	0	(0)	1	(1)
F	F	$Act2$	0.5	(0.5)	0.3	(0.3)	0.5	(0.5)
F	T	$Act2$	0.3	(0.3)	0.5	(0.5)	0.3	(0.3)
F	F	$Act1$	0	(0)	1	(0)	1	

- For $\Pi ID1$, we should first compute $\Pi(Pa'(V)|d_{ij},E)$, i.e. $\Pi(a|d,C=T)$ using the qualitative possibilistic adaptation of junction tree propagation algorithm (presented in Section 6.2.1) since the resulted possibilistic network is multiply connected (see Fig. 2). The first step of this phase is to build the junction tree $\mathcal{JT}$ as presented in Fig. 3 ($\mathcal{JT}$ contains 5 clusters $(AVD, BAV, ABC, BCD, CDV)$ and 4 separators (AV, AB, BC, CD)). Then, the global propagation step provides the following values:

$$\Pi(A=T|D=Act1, C=T) = 1,$$
$$\Pi(A=F|D=Act1, C=T) = 0.3 = 0.3,$$
$$\Pi(A=T|D=Act2, C=T) = 0.9 = 0.9,$$
$$\Pi(A=F|D=Act2, C=T) = 0.3 = 0.$$

Finally, these values are used to apply Eq. (19) as follows:

$$\min(\Pi_N(v=T|A=T, D=Act1), \Pi(A=T|D=Act1, C=T)) = \min(1/3, 1) = 1/3,$$
$$\min(\Pi_N(v=T|A=F, D=Act1), \Pi(A=F|D=Act1, C=T)) = \min(1, 0.3) = 0.3,$$
$$\min(\Pi_N(v=T|A=T, D=Act2), \Pi(A=T|D=Act2, C=T)) = \min(1, 0.9) = 0.9,$$
$$\min(\Pi_N(v=T|A=F, D=Act2), \Pi(A=F|D=Act2, C=T)) = \min(0, 0.3) = 0.$$

Thus, we can conclude that the optimal decision $D^* = Act_2$ with a maximal expected utility equal to 0.9.

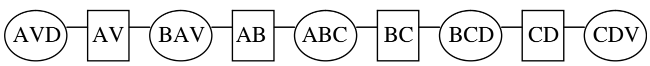

Fig. 3. The junction tree relative to the DAG of Fig. 2.

Table 11
Computation of the MEU according to the type of the DM

A	D	Pessimistic DM	Optimistic DM	Neutral DM
F	$Act2$	0.3	0	0
T	$Act2$	0.9	0.5	0.9
F	$Act1$	0.3	0.3	0.3
T	$Act1$	0	0.9	0

- For $\Pi ID2$, we should first compute $\Pi(V = T|Pa(V), d_{ij}, E)$, i.e. $\Pi(V = T|a, C = T)$ using the qualitative possibilistic adaptation of junction tree propagation algorithm (see Section 6.2.1). The junction tree $\mathcal{JT}$ is similar to the one obtained in the case of ordinal utilities (see Fig. 3) Different values of MEU according to the type of DM are presented in Table 11. From Table 11 it is clear that if the decision maker is pessimistic (resp. optimistic, neutral) then, the optimal action D^* is $Act2$ (resp. $Act1, Act2$) with a maximal expected utility equal to 0.9.

7. Possibilistic influence diagram toolbox (PIDT)

To illustrate our work, we have implemented a "possibilistic influence diagrams toolbox" (PIDT) with Matlab 6.5. Fig. 4 represents the architecture of PIDT. First, we should provide a file describing the PID, i.e.:

- parameters relative to the DAG structure, i.e. for each node, its name, its type (chance, decision or value node), its cardinality and its parent set;
- parameters relative to the quantification of the network, i.e. initial conditional possibility distributions of chance nodes in the context of their parents and utility functions for value nodes;

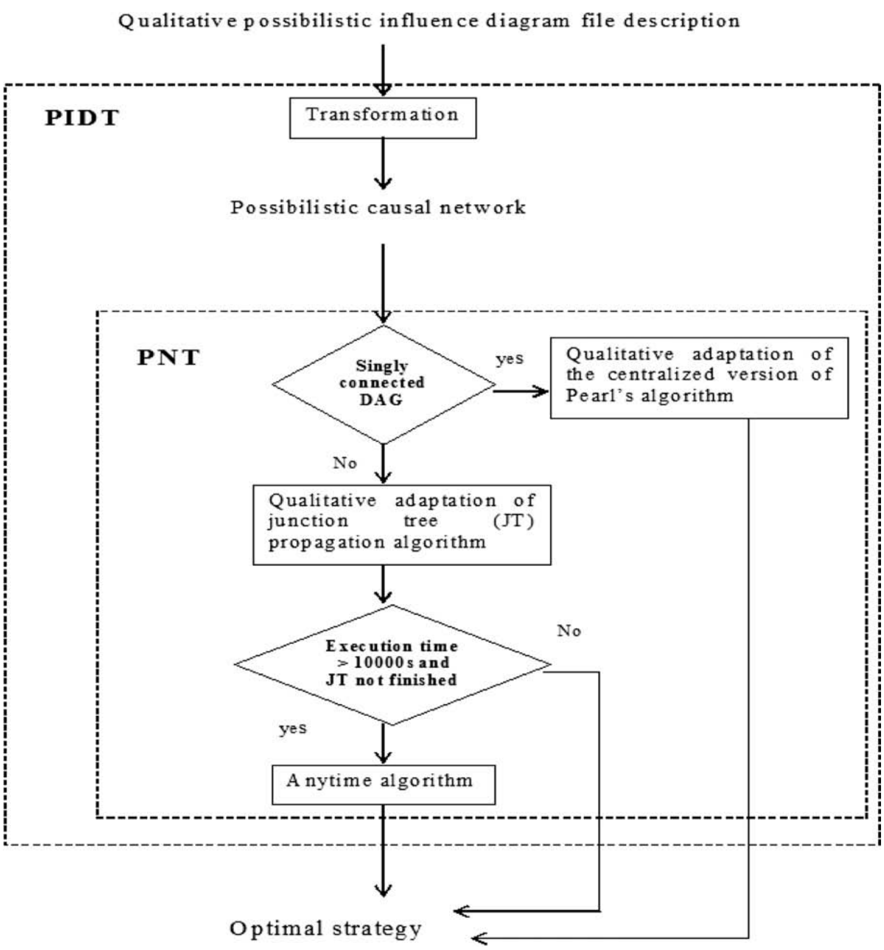

Fig. 4. PIDT architecture.

- if we use binary utilities, we should specify the decision maker behavior (optimistic, pessimistic or neutral) and his utility scale;
- the eventual evidence on chance nodes.

Given these parameters, the PIDT will proceed to the transformation of the possibilistic influence diagram into a possibilistic network and the application of the appropriate propagation algorithm using *Possibilistic Networks Toolbox* (PNT) which is a possibilistic toolbox implementing different propagation algorithms for min-based and product-based possibilistic networks (Ben Amor et al., 2001).

Since we treat qualitative possibilistic influence diagrams, their transformation leads to qualitative possibilistic networks. Then, if we are in the presence of a singly connected DAG, then the qualitative possibilistic adaptation of the centralized version of Pearl's algorithm is used. Otherwise, i.e. if the DAG is multiply connected, then the qualitative possibilistic adaptation of junction trees algorithm is used. In the case that this algorithm fails (i.e. time execution >10,000 s), then the anytime algorithm is used. This algorithm has the advantage to avoid eventual blocking especially with networks having complex structure with a great number of nodes. Finally, the PIDT generates the optimal strategy maximizing the expected utility.

8. Conclusion

This paper proposes a new approach for decision making under uncertainty using influence diagrams in the possibilistic framework.

The possibility theory encodes uncertainty in two ways, quantitatively using numerical values in the unit interval or qualitatively using a total preferential relation between different states of the world. These two interpretations lead to several possibilistic influence diagrams organized in three principal classes. Namely, product-based (quantitative possibilistic influence diagrams), min-based (qualitative possibilistic influence diagrams) and mixed possibilistic influence diagrams.

In this paper, we have studied qualitative ones, i.e. where the quantification of chance nodes is done via qualitative possibilistic distributions. For value nodes, we have proposed two ways for its quantification: in the first one we used ordinal utilities (Neumann and Morgenstern, 1948) and in the second qualitative binary utilities (Giang and Shenoy, 2005) are used.

To evaluate qualitative possibilistic influence diagrams, we have proposed an indirect method based on two phases: first we will transform the initial network into a possibilistic network (Ben Amor et al., 2001), then, we will apply propagation mechanism on this secondary structure.

The principle of the transformation phase is to transform decision and value nodes into chance nodes which will be quantified by conditional possibility distributions. Indeed, utility function is converted into a possibility distribution. This conversion depends on the type of utility used to quantify the value node in the possibilistic influence diagram.

Our developments show that quantitative possibilistic influence diagrams are very close to standard ones, in fact they share same operators (i.e. product operator). This is not the case for qualitative possibilistic influence diagrams, since the min operator has different properties than the product one which leads to a deep study of normalization constraints. In addition, value node transformation procedure in quantitative possibilistic influence diagrams is quiet similar to the probabilistic one used for standard influence diagrams, whereas it is different in qualitative influence diagrams.

The evaluation of standard influence diagrams is known to be an NP-hard problem (Cooper, 1988). This complexity remains the same in the possibilistic framework since the transformation phase is linear while the propagation phase is NP-hard (Ben Amor et al., 2001).

Nevertheless, our evaluation method has the advantage to avoid eventual blocking especially with networks having complex structure with a great number of nodes. In fact, once the transformation done, we try to apply an exact propagation algorithm, if it fails (i.e. time execution >10,000 s) we switch to the anytime algorithm (Ben Amor et al., 2001) which progressively converges to exact marginals. This algorithm is polynomial and it provides a high number of exact marginals (i.e. 96.42%).

The proposed approach, has been implemented in a possibilistic influence diagram toolbox (PIDT) which can be seen as a decision support system.

A direct improvement of our approach will be to extend the proposed evaluation methods to deal with more than one value node in order to treat multi-objective decision problems. Our idea is to consider decision nodes sequentially as proposed by Zhang's method (Zhang et al., 1994; Zhang, 1998) in the standard case. This method is based on previous advances made by Zhang et al. (1994) regarding the notion of stepwise decomposable influence diagrams and Bayesian inference techniques. The principle is to decompose the influence diagram into two components called tail and body with respect to the tail decision node, i.e. the last one in the ordered list of the whole decision nodes. Then the tail is evaluated separately as a possibilistic influence diagram with a unique value node and its optimal decision rule is merged with the body which will be considered as the new influence diagram to evaluate. This process is repeated iteratively until the body contains one value node. The decomposition theorem used in this method is based on Baye's theorem, the main challenge in the possibilistic framework will be to define the suitable decomposition theorem regarding the interpretation of the possibilistic scale.

Another interesting line of research will be to extend our work to mixed influence diagrams in order to treat the case where experts and decision makers are heterogeneous, i.e. they express their uncertainty in both qualitative and quantitative setting.

References

Ben Amor, N., Benferhat, S., Mellouli, K., 2001. Anytime propagation algorithm for min-based possibilistic graphs. In: Soft Computing a Fusion of Foundations Methodologies and Applications, vol. 8. Springer Verlag, pp. 50–161..

Ben Amor, N., 2002. Qualitative possibilistic graphical models: From independence to propagation algorithms. Ph.D. dissertation.

Borgelt, C., 1998. J. Gebhardt Rudolf Kruse. Possibilistic graphical models. In: Proceedings of International School for the Synthesis of Expert Knowledge (ISSEK'98), Udine, Italy.

Cooper, G.F., 1988. A method for using belief networks as influence diagrams. In: Fourth Workshop on Uncertainty in Artificial Intelligence, pp. 55–63.

Dubois, D., Prade, H., 1988. Possibility Theory an Approach to Computerized Processing of Uncertainty. Plenum Press, New York, NY.

Dubois, D., Gogo, L., Prade H., Zapico, A., 1998. Making decision in a qualitative setting: From decision under uncertainty to case-based decision. In: Proceedings of Principles of Knowledge, Representation and Reasoning (KR'98), pp. 594-607, Trento (Italia).

Dubois, D., Prade, H., Sabbadin, R., 2001. Decision theoretic foundation of qualitative possibility theory. European Journal of Operational Research 128, 478–495.

Fonck, P., 1994. Réseaux d'inférence pour le raisonnement possibiliste. PhD thesis, Université de Liège, Faculté des Sciences.

Garcia, L., Sabbadin, R., 2006. Possibilistic influence diagrams. In: 17th European Conference on Artificial Intelligence (ECAI'2006), Italy, ECAI.

Giang, P.H., Shenoy, P.P., 2005. Two axiomatic approaches to decision making using possibility theory. European Journal of Operational Research 162, 450–467.

Guezguez, W., Ben Amor, N., Mellouli, K., 2006. Qualitative possibilistic influence diagrams. In: 14th International Enformatika Conference. Transaction on Engineering, Computing and Technology, Prague, vol. 14, pp. 430–435.

Howard, R.A., Matheson, J.E., 1984. Influence diagrams. In: R.A. Howard, J.E. Matheson (Eds.), The Principles and Applications of Decision Analysis, vol. 2. Strategic Decisions Group, Menlo Park, CA.

Huangn, C., Darwiche, A., 1994. Inference in belief networks: A procedural guide. International Journal of Approximate Reasoning 11, 1–158.

Jensen, F.V., 1996. An Introduction to Bayesian Networks. Taylor and Francis, London United Kingdom.

Jensen, F.V., 2002. Bayesian Networks and Decision Graphs. Springer, Statistics for Engineering and Information Science.

Kim, J., Pearl, J., 1987. Convice: A conversational inference consolidation engine. IEEE Transactions on Systems, Man and Cybernetics 132, 7–120.

Kjaerulff, U., 1990. Triangulation of graphs-algorithms giving small total state space. Technical report.

Neumann, J.V., Morgenstern, O., 1948. Theory of Games and Economic Behavior. Princeton University Press.

Pearl, J., 1986. Fusion propagation and structuring in belief networks. Artificial Intelligence 29, 241–288.

Pearl, J., 1988. Probabilistic Reasoning in Intelligent Systems: Networks of Plausible Inference. Morgan Kaufmman, San Francisco, CA.

Sanchez, D.G., Druzdzel, M.J., 2004. An efficient sampling algorithm for influence diagrams. In: Proceedings of the Second European Workshop on Probabilistic Graphical Models (PGM), pp. 97–104, The Netherlands.

Savage, L.J., 1972. The Foundations of Statistics. Dover, New York.

Shachter, R.D., 1986. Evaluating influence diagrams. Operation Research 34, 871–882.

Shachter, R.D., Poet, M.A., 1992. Decision making using probabilistic inference methods. In: Proceedings of 8th Conference on Uncertainty in Artificial Intelligence, pp. 276–283.

Shenoy, P.P., 1994. A comparison of graphical techniques for decision analysis. European Journal of Operational Research 78, 1–21.

Tatman, J.A., Shachter, R.D., 1990. Dynamic programming and influence diagrams. IEEE Transactions on Systems, Man and Cybernetics 20, 365–379.

Whalen, T., 1984. Decision making under uncertainty with various assumptions about available information. IEEE Transactions on Systems, Man and Cybernetics 14, 888–900.

Xiang, Y., Ye, C., 2001. A simple method to evaluate influence diagrams. In: Published in Third International Conference on Cognitive Science.

Yager, R.R., 1979. Possibilistic decision making. IEEE Transactions on Systems, Man and Cybernetics 9, 388–392.

Zadeh, L., 1978. Fuzzy sets as a basis for a theory of possibility. Fuzzy Sets and Systems 1, 3–28.

Zhang, N.L., Qi, R., Pool, D., 1994. A computational theory of decision networks. International Journal of Approximate Reasoning 7.

Zhang, N.L., 1998. Probabilistic inference in influence diagrams. In: Proceedings of 14th Conference on Uncertainty in Artificial Intelligence, pp. 514–522.